Computers & Industrial Engineering 110 (2017) 424–435

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A survey and classification of Opposition-Based Metaheuristics

Nicolás Rojas-Morales *, María-Cristina Riff Rojas, Elizabeth Montero Ureta

Universidad Técnica Federico Santa María, Avenida España 1680, Valparaíso, Chile

ARTICLE INFO

Article history:
Received 2 December 2016
Received in revised form 10 June 2017
Accepted 16 June 2017
Available online 19 June 2017

Keywords:
Opposition-Based Learning
Metaheuristics

ABSTRACT

Opposition-Based Learning (OBL) is a research area that has been widely applied in several algorithms for improving the search process. In this work we present a revision of several applications of OBL in metaheuristics and some metaheuristic approaches that are inspired in OBL. For reviewing each OBL approach we analyze the objective of including OBL, the role performed by the OBL component, the type of OBL and the type of problem tackled. We also propose a classification of these approaches that apply or are inspired in OBL. Our goal is to motivate researchers in metaheuristics to include ideas from OBL and report which strategies were successfully applied.

1. Introduction

A metaheuristic can be defined as an iterative generation process which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space (Osman & Laporte, 1996). In general, metaheuristics identify promising regions of the search space and exploit them to obtain the best quality solutions from those regions. However, metaheuristics based approaches present typical difficulties: getting trapped in local optima, have convergence problems, among others. For this reason, the design of strategies for improving the search of metaheuristics has been studied for several years (Talbi, 2002, 2009).

Opposition-Based Learning (OBL) is a concept originally introduced in 2005 by Tizhoosh (2005), related to obtaining complementary candidates from a set of solutions. This relationship is defined as "opposite" and it considers to map candidate solutions for increasing the coverage of the search space, accuracy and convergence of the search process (Malisia, 2008). The objective is to support the search process of an original algorithm considering more candidate solutions, that are opposite solutions.

In the metaheuristics community, many Opposition-Based metaheuristics have been reported. In Section 3, we present an extensive revision of OBL ideas included on metaheuristics and some OBL inspired approaches. In Section 4, we propose a classification of OBL approaches in metaheuristics, followed by a discussion on Section 5 and some conclusions on Section 6. In the following section we formally define different types of OBL.

2. Opposition-based learning

There are several definitions of what *opposition* is, considering a long list of situations and contexts where this concept can be applied. Focused on metaheuristics and optimization, *opposition* has been presented as a relationship between a pair of candidate solutions. Both in combinatorial and continuous optimization, each candidate solution has its own particularly defined opposite candidate. In general, there are two ways of searching using opposite candidate solutions: defining a function for mapping every solution of the search space with its own opposite solution or, searching for solutions with opposite quality.

Lets suppose that we are interested in solving a problem whose variable domains are defined in $\mathcal{C}$ space, where $\mathcal{C}$ can be numerical or categorical. Also, $X = [x_1, x_2, \ldots, x_M]$ is a point in a M-dimensional space where $x_i \in [a_i, b_i], \forall i \in [1, M]$. Next, we will present different existing types of OBL to obtain opposite candidate solutions $\breve{X}$ that consider variables in a M-dimensional space. More specifically, $M \in [1, \infty)$.

Definition 1 (*Type-I Opposition*). Given $\Phi : \mathcal{C} \rightarrow \mathcal{C}$ an opposition mapping function that maintains the characteristics of the input space. Thus, a Type-I Opposite candidate solution is defined by $\breve{X} = \Phi(X)$

Here, Φ is a function that maps every solution X in the search space to their opposite solution $\breve{X}$. Fig. 1 shows a search space example of a problem to be solved. There are seven regions named from A to G, where the darker the region the better the quality solution. Suppose that a candidate solution X is in region D, where worst quality solutions are placed. Using the mapping function Φ, we can obtain its opposite candidate solution $\Phi(X) = \breve{X} = -X$. This

* Corresponding author.
E-mail addresses: nicolasrojas@acm.org (N. Rojas-Morales), maria.cristina.riff@gmail.com (M.-C. Riff Rojas), elizabeth.montero@gmail.com (E. Montero Ureta).

http://dx.doi.org/10.1016/j.cie.2017.06.028
0360-8352/

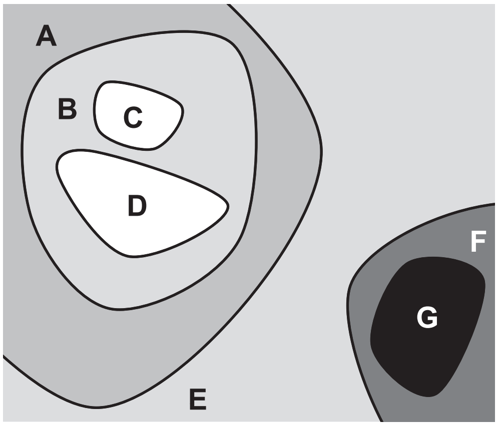

Fig. 1. Search space example.

solution should be in other region of the search space, for example in Fig. 1, in region B. The key idea is to generate more candidate solutions using Φ and, continue the search process with the higher quality solutions. It is important to emphasize that Φ is defined as a one-to-one mapping function. Also, the relationship between two candidate solutions is symmetric.

From Type-I Opposition, there are two more extensions that are defined according to the distance from $\breve{X}$: *Type-I Quasi-Opposition* and *Type-I Super-Opposition* (Tizhoosh, Ventresca, & Rahnamayan, 2008).

Definition 2 (*Type-I Quasi-Opposition*). Given $X \in \mathcal{C}$ and d a distance function. We define that points $\breve{X}_q$ are Type-I Quasi-opposite of X when $d(\breve{X}_q, X) < d(\breve{X}, X)$.

Definition 3 (*Type-I Super-Opposition*). Given $X \in \mathcal{C}$ and d a distance function. We define points $\breve{X}_s$ are Type-I Super-opposite of X when $d(\breve{X}_s, X) > d(\breve{X}, X)$.

Quasi-opposite points $\breve{X}_q$ are defined closer to the original candidate solution X than a Type-I Opposition point $\breve{X}$. The hypothesis is that quasi-opposite points have a higher chance to be closer to an optimal solution than opposite points. On the other hand, Super opposite candidate solutions $\breve{X}_s$ are farther to a Type-I Opposition point $\breve{X}$.

Other type of opposition defines a relationship between a pair of candidate solutions considering an evaluation function for mapping them:

Definition 4 (*Type-II Opposition*). Given $X \in \mathcal{C}$ and $\Upsilon : f(\mathcal{C}) \to f(\mathcal{C})$ by a Type-II Opposition mapping where f is a defined performance function. Then, the set of Type-II opposites of X are completely defined by Υ.

In general, Υ maps a candidate solution considering the extreme values of a defined evaluation function. As Fig. 1 shows, suppose that X is in region E and its opposite $\Upsilon(X) = -X$ is in region D, where $f(X) > f(-X)$ in a maximization scenario. The *Type-II Opposition* strategy has been used for exploring the search space and temporary discard regions that are possibly related to low quality solutions.

Generalized OBL (GOBL) (Wang, Wu, Rahnamayan, & Kang, 2009) is another version of OBL, originally presented as *Space Transformation Search* (STS) (Wang et al., 2009). Similarly to the Quasi-Opposition idea, the aim of GOBL was to obtain candidate solutions closer to the global optimum.

Definition 5 (*Generalized OBL*). Given $X = [x_1, \dots, x_M]$ a solution in a M-dimension space. An opposite candidate solution $\breve{X}_g = [\breve{x}_1, \dots, \breve{x}_M]$ is defined by a weight parameter k, that controls how close their respective dimensions are. Each dimension j of X is mapped using:

$$\breve{x}_j = k * (a_j + b_j) - x_j \tag{1}$$

where k is a random number $\in [0, 1]$ and a_j and b_j are the minimum and maximum values for jth dimension of X.

Center-Based Sampling (CBS) (Rahnamayan & Wang, 2009) is a variant of OBL similar to GOBL. Here, the objective is to obtain opposite candidate solutions closer to the center of the domain of each variable.

Definition 6 (*Center-Based Sampling*). Given $X = [x_1, \dots, x_M]$ a solution in a M-dimension space. An opposite candidate solution $\breve{X}_c = [\breve{x}_1, \dots, \breve{x}_M]$ is defined by a random point between X and its opposite point $\breve{X}$. Each dimension j of X is mapped using:

$$\breve{x}_j = rand_j * (a_j + b_j - 2 * x_j) + x_j \tag{2}$$

where $rand_j$ is a uniformly distributed random number $\in [0, 1], j \in [1, M]$ and a_j and b_j are the minimum and maximum values of the jth component of X.

In literature there are other variants of OBL: *Quasi-Reflection OBL* (QR_{OBL}) (Ergezer, Simon, & Du, 2009) and *OBL using the Current Optimum* (CO_{OBL}) (Xu, Wang, He, & Wang, 2011). Table 1 summarizes all these types of OBL. For each type, we present the opposite candidate of a solution X and the required inputs for obtaining this opposite candidate solution.

Fig. 2 shows a minimization problem example that considers one dimension solutions ($M = 1$) and all the OBL types described. Let's suppose that $\mathcal{C}$ space is $\mathcal{R}$ and the candidate solution $x \in [0, 10]$, specifically $x = 2$. Also, let's suppose that the current best solution found is $x_{co} = 1$. Considering the definitions proposed in Tizhoosh et al. (2008), the Type-I opposite $\breve{x}$ will be equal to 8, the Type-I Quasi opposite $\breve{x}_q = 6$ and the Type-I Super opposite $\breve{x}_s \in (8, 10]$. Then, as $\breve{x}_{qr}$ is a reflection of $\breve{X}_q$ across the center of the domain, $\breve{x}_{qr} = 4$. About CBS, $\breve{x}_c$ can be between 4 and 6, near

Table 1
OBL types summary.

OBL type	Opposite of X	Who can be their opposite $\breve{X}$	Input
Type-I Opposition	$\breve{X}$	Solution between two defined boundaries	–
Type-I Quasi-Opposition	$\breve{X}_q$	Solution closer to X than $\breve{X}$	$\breve{X}$, Distance Metric
Type-I Super-Opposition	$\breve{X}_s$	Solution farther to X than $\breve{X}$	$\breve{X}$, Distance Metric
Type-II Opposition	$\breve{X}_{II}$	Solution with opposite performance	–
Generalized OBL	$\breve{X}_g$	Solution can be nearer to X than $\breve{X}$	k parameter
Quasi-Reflection OBL	$\breve{X}_{qr}$	Reflection from the center of the domain of the $\breve{X}_q$	$\breve{X}_q$
Center-Based Sampling	$\breve{X}_c$	Solution close to the center of the domain	k_1 and k_2 parameters
OBL using Current Optimum	$\breve{X}_{CO_{OBL}}$	Solution related to the current optimum (X_{co}) or same as GOBL	$X_{co}, rand(0,1)$