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Some new ranking criteria in data envelopment analysis under uncertain environment

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ABSTRACT

Data Envelopment Analysis (DEA) is a very effective method to evaluate the relative efficiency of decision-making units (DMUs), which has been applied extensively to education, hospital, finance, etc. However, in real-world situations, the data of production processes cannot be precisely measured in some cases, which leads to the research of DEA in uncertain environments. This paper will give some researches to uncertain DEA based on uncertainty theory. Due to the uncertain inputs and outputs, we will give three uncertain DEA models, as well as three types of fully ranking criteria. For each uncertain DEA model, its crisp equivalent model is presented to simplify the computation of uncertain models. Finally, a numerical example is presented to illustrate the three ranking criteria.

1. Introduction

Data envelopment analysis (DEA), as an useful management and decision tool, has been widely used since it was first invented by Charnes, Cooper, and Rhodes (1978). The method is followed by a series of theoretical extensions, such as Banker, Charnes, and Cooper (1984), Charnes, Cooper, Golany, Seiford, and Stutz (1985), Petersen (1990), Tone (2001) and Cooper, Seiford, and Tone (2000). More DEA papers can refer to Seiford (1994) in which 500 references are documented.

In many cases, decision makers are interested in a complete ranking over the dichotomized classification. The researches on ranking have come up for this reason. Over the last decade, many literatures on ranking in DEA have been published. By evaluating DMUs through both self and peer pressure, Sexton, Silkman, and Hogan (1986) can attain a more balanced view of the decision-making units. Andersen and Petersen (1993) developed the super-efficiency approach to get a ranking value which may be greater than one through evaluated DMU's exclusion from the linear constraints. In the benchmark ranking method (Torgersen, Forsund, & Kittelsen, 1996), a DMU is highly ranked if it is chosen as a useful target for many other DMUs.

Most methods of ranking DMUs assume that all inputs and outputs data are exactly known. However, in real situations, such as in a manufacturing system, a production process or a service system, inputs and outputs are volatile and complex so that they are difficult to measure in an accurate way. Thus, people tend to use fuzzy theory to describe the indeterminate inputs and outputs, which motivates the fuzzy DEA. Generally speaking, fuzzy DEA method can be catagorized into four types: the tolerance approach, the α-level based approach, the fuzzy ranking approach and the possibility approach (Adel, Emrouznejad, & Tavana, 2011). In the tolerance approach (refer to Sengupta (1992)), tolerance levels on constraint violations are defined to integrate fuzziness into the DEA models, and the input and output coefficients can be thus treated as crisps. The α-level based approach may be the most popular model of fuzzy DEA. This method discretize the original problem into a series of parametric programs in order to decide the α-cuts of the membership function of efficiency. Related studies include Kao and Liu (2000), Entani, Maeda, and Tanaka (2002), Liu (2008) and Angiz, Emrouznejad, and Mustafa (2012), etc. The fuzzy ranking model is first proposed by Guo and Tanaka (2001), and it focus on determining the fuzzy efficiency scores of DMUs using optimization methods which require ranking fuzzy sets. One can also refer to León, Liern, Ruiz, and Sirvent (2003), Wang and Luo (2006) or Angiz, Tajaddini, Mustafa, and Kamali (2012) for more concepts and information of the fuzzy ranking method. In the possibility approach, the fuzzy DEA models are converted to possibility linear program problem by using possibility

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measures. See Lertworasirikul, Fang, Joines, and Nuttle (2003) for example. Other studies on fuzzy DEA include fuzzy goal programming method (Sheth & Konstantinos, 2003), fuzzy random DEA model in hybrid uncertain environments (Qin & Liu, 2010), fuzzy rough DEA model (Shiraz, Charles, & Jalalzadeh, 2014), cross evaluation approach (Costantino, Dotoli, Epicoco, Falagario, & Sciancalepore, 2012), and fuzzy clustering approach (David & Deep, 2012), etc.

Although the fuzzy DEA models are popular and in most time effective, it may bring some problems to the decision makers in some cases. This is because the possibility measure defined in fuzzy theory doesn't satisfy duality, as explained in Liu (2012). For this reason, an uncertainty theory was founded by Liu (2007) in 2007, and refined by Liu (2010a) in 2010 to deal with the people's belief degree mathematically. A concept of uncertain variable is used to model uncertain quantity, and belief degree is regarded as its uncertainty distribution. As extensions of uncertainty theory, uncertain programming was proposed by Liu (2009) in 2009, which aims to deal with the optimal problems involving uncertain variable. Since then, uncertainty theory was used to solve a variety of real optimal problems, including finance (Chen & Liu, 2010; Peng & Yao, 2010; Liu, 2013), reliability analysis (Liu, 2010b; Zeng, Wen, & Kang, 2013), uncertain graph (Gao, 2013; Gao & Gao, 2013), etc. As an application, this work was followed by uncertain multiobjective programming models, uncertain goal programming models (Liu & Chen, 2013), and uncertain multilevel programming models (Liu & Yao).

In this paper, we will assume the inputs and outputs in DEA models are uncertain variables, and introduce some new DEA models and their ranking criteria based on uncertainty theory. The remainder of this paper is organized as follows: Some basic concept and results on uncertainty theory will be introduced in Section 2; Section 3 will give some basic introduction to DEA models; The method to obtain uncertainty distribution is introduced in Section 4. In Section 5, we will give three uncertain DEA models, three fully ranking criteria, as well as their equivalent deterministic models. Finally, a numerical example will be given to illustrate the uncertain DEA model and the ranking method in Section 6.

2. Preliminaries

Uncertainty theory was founded by Liu (2007) in 2007 and refined by Liu (2010a) in 2010. Nowadays uncertainty theory has become a branch of axiomatic mathematics for modeling human uncertainty. In this section, we will state some basic concepts and results on uncertain variables. These results are crucial for the remainder of this paper.

Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is assigned a number $\mathcal{M}\{\Lambda\} \in [0,1]$. In order to ensure that the number $\mathcal{M}\{\Lambda\}$ has certain mathematical properties, Liu (2007, 2010a) presented the three axioms:

(i) $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ.
(ii) $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.
(iii) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \leqslant \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$$

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an uncertainty space. In order to obtain an uncertain measure of compound event, a product uncertain measure was defined by Liu (2012), thus producing the fourth axiom of uncertainty theory:

(iv) Let $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$ be uncertainty spaces for $k = 1, 2, \ldots, \infty$. Then the product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty}\Lambda_k\right\} = \bigwedge_{k=1}^{\infty}\mathcal{M}_k\{\Lambda_k\}.$$

An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers (Liu, 2007). In order to describe an uncertain variable in practice, the concept of uncertainty distribution is defined as

$$\Phi(x) = \mathcal{M}\{\xi \leqslant x\} \tag{1}$$

for any real number x. For example, the linear uncertain variable $\xi \sim \mathcal{L}(a,b)$ has an uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leqslant a \\ (x-a)/(b-a), & \text{if } a \leqslant x \leqslant b \\ 1, & \text{if } x \geqslant b. \end{cases} \tag{2}$$

An uncertain variable ξ is called zigzag if it has a zigzag uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leqslant a \\ (x-a)/2(b-a), & \text{if } a \leqslant x \leqslant b \\ (x+c-2b)/2(c-b), & \text{if } b \leqslant x \leqslant c \\ 1, & \text{if } x \geqslant c \end{cases} \tag{3}$$

denoted by $\mathcal{Z}(a,b,c)$ where $a,\ b,\ c$ are real numbers with $a < b < c$. An uncertain variable ξ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e-x)}{\sqrt{3}\sigma}\right)\right)^{-1} \tag{4}$$

denoted by $\mathcal{N}(e,\sigma)$ where e and σ are real numbers with $\sigma > 0$. An uncertainty distribution Φ is said to be regular if its inverse function $\Phi^{-1}(\alpha)$ exists and is unique for each $\alpha \in (0,1)$. The uncertain variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{n}(\xi_i \in B_i)\right\} = \bigwedge_{i=1}^{n}\mathcal{M}\{\xi_i \in B_i\} \tag{5}$$

for any Borel sets $B_1, B_2, \ldots, B_n$.

Theorem 1 (Liu, 2010a). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If f is a strictly increasing function, then*

$$\xi = f(\xi_1, \xi_2, \ldots, \xi_n) \tag{6}$$

is an uncertain variable with inverse uncertainty distribution

$$\Psi^{-1} = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \ldots, \Phi_n^{-1}(\alpha)). \tag{7}$$

Theorem 2 (Liu & Ha, 2010). *Assume $\xi_1, \xi_2, \ldots, \xi_n$ are independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If $f(x_1, x_2, \ldots, x_n)$ is strictly increasing with respect to $x_1, x_2, \ldots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \ldots, x_n$, then the uncertain variable $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$ has an expected value*

$$E[\xi] = \int_0^1 f(\Phi_1^{-1}(\alpha), \ldots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \ldots, \Phi_n^{-1}(1-\alpha))\mathrm{d}\alpha \tag{8}$$

provided that $E[\xi]$ exists.