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A bi-level programming approach for production-distribution supply chain problem

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ABSTRACT

This paper investigates a decentralized production-distribution supply chain consisting of one manufacturer and one distributor where the demand is jointly influenced by pricing and advertising policies. The control of decision variables is partitioned amongst the members. The manufacturer decides about wholesale price, production interval, expenditure of global advertising, and participation rate in the distributor's local advertising expenditure while the decisions on selling price, local advertising expenditure, and allocation of demands to depots are made by the distributor. We propose a Stackelberg game framework and develop two nonlinear bi-level programming models by switching the leader and follower roles between the manufacturer and the distributor. Four hierarchical solution algorithms are proposed by combining genetic algorithm and particle swarm optimization to tackle the bi-level programming models. Finally, computational experiments are carried out to analyze and compare the efficiency and effectiveness of the proposed solution procedures.

1. Introduction

The last two decades have witnessed a considerable interest by researchers in supply chain management (SCM) and a new emphasis on the interactions among supply chain members. There are two approaches towards decision making in supply chain: centralized and decentralized. In a centralized supply chain, an individual member with dominant position delineates the policies for other members. In such a case, members are willing to cooperate based on the declared policies. In decentralized supply chain, however, firms do not tend to cooperate and seek to optimize their own objectives rather than those of the entire system.

Bi-level programming is an appropriate technique for modeling decentralized management problems with two non-cooperative decision makers in a hierarchical structure. From a historical point of view, bi-level programming as a special case of multi-level optimization is closely related to the economic problem of Stackelberg (Von Stackelberg, 1952) in the field of game theory.

The decision maker at the upper level of the hierarchy is known as *leader*, while the lower level decision maker is termed as *follower*. Each decision maker controls a set of decision variables subject to a set of constraints and aims to optimize his own objective function. The actions taken by the leader affect the reactions that the follower will take, as a response to the leader's decisions. Therefore, the actions of one influence the choices and payoffs available for the other.

The general form of a bi-level programming problem (BLPP) is stated as follows:

$$\begin{aligned} &\min_{x\in X} && F(x,y) \\ &\text{subject to} && G(x,y) \leqslant 0 \\ & && \min_{y\in Y} \quad f(x,y) \\ & && \text{subject to} \quad g(x,y) \leqslant 0 \end{aligned}$$

where $x \in X \subseteq \mathcal{R}^n$ is the set of leader's decision variables and $y \in Y \subseteq \mathcal{R}^m$ is the set of follower's decision variables. Similarly, the functions $\mathrm{F} : \mathcal{R}^{n\times m} \to \mathcal{R}$ and $\mathrm{f} : \mathcal{R}^{n\times m} \to \mathcal{R}$ are objective functions of leader and follower, while $\mathrm{G} : \mathcal{R}^{n\times m} \to \mathcal{R}^{p}$ and $\mathrm{g} : \mathcal{R}^{n\times m} \to \mathcal{R}^{q}$ are upper and lower level constraints, respectively.

The relaxed feasible region (or constraint region) of BLPP is:

$$S = \{(x,y) : x \in X,\ y \in Y,\ G(x,y) \leqslant 0 \text{ and } g(x,y) \leqslant 0\}$$

For a fixed $x \in X$, the *lower-level feasible set* is defined by:

$$S(x) = \{y \in Y : g(x,y) \leqslant 0\}$$

While the lower-level reaction set (or rational reaction set) is:

$$R(x) = \{y \in Y : y \in \arg\min(f(x,\tilde{y}), \tilde{y} \in S(x))\}$$

Every $y \in R(x)$ is a rational response.

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Finally, the *induced region* (or *inducible region*) is defined as:

$$IR = \{(x,y) : (x,y) \in S,\ y \in R(x)\}$$

This set is formed by the feasible points of the BLPP, corresponds to the feasible set of the leader. It is usually non-convex and can even be disconnected or empty in presence of upper-level constraints. The BLPP is equivalent to optimizing the leader's objective F over the inducible region IR. A feasible point $(x^*, y^*) \in IR$ is a global optimum of BLPP (or *Stackelberg equilibrium*) if $F(x^*, y^*) \leqslant F(x, y)$ for all $(x, y) \in IR$.

Bi-level programming techniques have been widely applied to handle realistic decentralized decision problems. Gao and et al. (2011) develop two nonlinear bi-level programming models for pricing problem in a vendor-buyer supply chain. Sadigh, Mozafari, and Karimi (2012) address a multi-product manufacturer-retailer supply chain in which the demand of each product is jointly influenced by price and advertising expenditure. They apply bi-level programming approach and present several solution procedures to find optimal policies in terms of pricing, advertising and inventory management. Mokhlesian and Zegordi (2014) develop a nonlinear bi-level programming model for a multi-product two-echelon supply chain consisting of one manufacturer and multiple retailers. Zhang, Lu, and Gao (2015) establish a bi-level pricing and replenishment strategy optimization model in high-tech industry where the buyer and the vendor are respectively designated as the leader and the follower. Ma, Wang, and Zhu (2014) consider joint pricing and lot sizing problem in a two-echelon supply chain system with one manufacturer and one retailer. They establish two nonlinear bi-level programming models by switching the leader and the follower roles between the manufacturer and the retailer.

Although production-distribution supply chain problems have been topics of wide interest in SCM research, little attention has been given to analyzing the problems by means of bi-level programming approach. Ryu, Dua, and Pistikopoulos (2004) present a bi-level programming model to a production and distribution planning problem where the follower's problem can be modeled by linear programs. A similar production and distribution problem with probabilistic parameters is formulated as a bi-level linear multi-objective programming problem by Roghanian (Roghanian, Sadjadi, & Aryanezhad, 2007). Calvete, Galé, and Oliveros (2011) study a hierarchical production-distribution planning in which the distributor controls the design of the routes and the manufacturer controls the production process. In another paper (Calvete, Galé, & Iranzo, 2014), they address a mixed integer bi-level optimization model for the planning of a decentralized distribution network consisting of manufacturing plants, depots and customers.

This paper addresses a decentralized production-distribution supply chain consisting of one manufacturer and one distributor and develop two nonlinear bi-level programming models, one bi-level model considers the manufacturer as the leader (Manufacturer-Stackelberg), who has priority in deciding, and the other takes the distributor as the leader (Distributor-Stackelberg). Both the manufacturer and distributor aim to maximize their profits, but their decisions are related to each other in a hierarchical way. The distributor owns several depots to supply customers in different markets. The markets are also different in terms of size, geographic location, price elasticity of demand, and response to advertisement.

In order to boost the markets' demands, the distributor undertakes a given expenditure to conduct local advertising in each market. On the other hand, the manufacturer may in turn, set a budget for global advertising within the target markets. We assume that the manufacturer participates in a cooperative advertising program and undertakes a portion of distributor's local advertising expenditure. Cooperative advertising is seen to be among those topics intensely investigated in the literatures during the recent years (Aust & Buscher, 2014). This concept represents a financial agreement between the manufacturer and the distributor to share advertising expenditure based on which the manufacturer undertakes part of local advertising expenditure (participation rate) incurred by the distributor (Koh, 2013).

In the proposed BLPP, the control of decision variables is partitioned amongst the manufacturer and the distributor. The manufacturer decides about wholesale price, production (replenishment) interval, global advertising expenditure, and participation rate in the distributor's local advertising expenditure. The distributor, however, makes decisions on selling price, local advertising expenditure in each market, and allocation of markets' demands to depots in a way that total transportation cost is minimized. In fact, neither the manufacturer nor the distributor has direct control over the decision-making process of the other, but their decisions affect the other member's subsequent reactions. With the assumption of perfect information to be available about the follower, in both models, the leader could anticipate the reaction of the follower and acts in a way that is optimal for him.

The first contribution of this paper is to model a decentralized production-distribution supply chain problem by using of bi-level programming, which allows us to take into account how decisions made at the distribution segment of the supply chain can affect and be affected by decisions made at the manufacturing segment.

Bi-level programming models are non-convex and strongly NP-hard, even in their simplest case in which all the functions involved are linear (Ben-Ayed & Blair, 1990; Hansen, Jaumard, & Savard, 1992). Thus, it is quite difficult to deal with nonlinear version of these models. Traditional approaches for solving BLPP can be roughly divided into the following categories (Dempe, 2002): (1) vertex enumeration methods, (2) decent algorithm, (3) methods based on Karush-Kuhn-Tucker conditions, and (4) penalty functions, etc. The properties such as continuity, convexity and differentiability are necessary when proposing these methods. Hence, they have limitations on solving BLPP. meta-heuristic algorithms including stochastic optimization techniques are recognized as useful tools for solving BLPP which do not necessarily satisfy the classical optimization assumptions. This has motivated researchers to apply meta-heuristic methods to tackle BLPP (Gao & et al., 2011; Khilwani & et al., 2008; Koh, 2007; Marinakis & Marinaki, 2008; Mokhlesian & Zegordi, 2014; Rajesh & et al., 2003; Sadigh et al., 2012; Sahin & Ciric, 1998).

Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) are the generic computational techniques espoused from the progression of biological life in the natural humanity (Khilwani & et al., 2008). The second contribution of this study is to develop four hierarchical algorithms by combining GA and PSO to tackle the proposed bi-level programming models. In addition to these algorithms, an exhaustive grid search is adopted for validation purposes.

The rest of this paper is organized as follows. In Section 2, two bi-level programming models (Manufacturer-Stackelberg & Distributor-Stackelberg) are developed by formulating the manufacturer and the distributor problems. Section 3 proposes four hierarchical procedures for solving the Stackelberg models. Section 4 is devoted to experimental results and finally Section 5 discusses the concluding remarks.

2. Bi-level programming models for production-distribution supply chain

In this section, we describe general characteristics of the production-distribution supply chain and formulate the bi-level programming models.