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Technical Note

A Note on "Two new approaches for a two-stage hybrid flowshop problem with a single batch processing machine under waiting time constraint"

Zewen Huang [a], Zhongshun Shi [b,*], Cheng Zhang [a], Leyuan Shi [a,b]

[a] Department of Industrial Engineering & Management, Peking University, Beijing 100871, China
[b] Department of Industrial & Systems Engineering, University of Wisconsin-Madison, WI 53706, USA

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ABSTRACT

The two-stage flow shop scheduling problem with a batch processing machine and limited waiting time was studied in the paper entitled "Two new approaches for a two-stage hybrid flowshop problem with a single batch processing machine under waiting time constraint" [Chung, T.-P., Sun, H., Liao, C.-J., 2016, *Computers* & *Industrial Engineering*, 10.1016/j.cie.2016.11.031]. In the original paper, this problem was formulated as a mixed-integer linear programming model. However, this model is not correct and the blocking time is confusing. In this note, a correct mixed-integer linear programming model is proposed and the confusing blocking time is discussed. Numerical experiments are conducted to illustrate the necessity of correction.

1. Introduction

Chung, Sun, and Liao (2016) studied the two-stage flow shop scheduling problem with a batch processing machine and limited waiting time to minimize the makespan. There are n jobs. Each job j has a released time r_j and a size s_j. In the first stage, the decision should be made on which batch each job should be assigned to in the batch processing machine. The sum of the job sizes in a batch must be less than or equal to the batch capacity B. The processing time of one batch is equal to the largest processing time of jobs assigned to this batch. In the second stage, the buffer size is unlimited and the buffer service rule is first in and first out (FIFO), i.e., jobs will be processed in the single machine by the same sequence as that in the batch processing machine. The waiting time of each job in the buffer is limited by an identical W. We denote this problem as $\beta \rightarrow \delta|limited\ waiting\ time, r_j, FIFO|C_{max}$, where β means the batch machine, δ means the single machine and C_{max} is the objective function. The medium term illustrates the above main constraints. This problem is common and has broad applications in industrial practice, such as wafer fabrication and heat-treating ovens. A mixed-integer linear programming model and two immunoglobulin-based artificial immune system algorithms are proposed in Chung et al. (2016).

However, after a careful examination of the proposed mathematical model and the algorithms, we found that the constraints about limited waiting time are not correct and the blocking time is confusing. The proposed heuristic algorithms are efficient. But since the proposed heuristic algorithms use the incorrect constraints to estimate the feasibility of each solution, the related computational results are also not accurate.

The remainder of this note is organized as follows. In Section 2, we present the mathematical model in Chung et al. (2016) and analyze the existing errors. In Section 3, we propose a correct mixed-integer linear programming model. In Section 4, we discuss the blocking time. Numerical experiments are conducted in Section 5 and conclusions are given in Section 6.

2. The formulation of Chung et al. (2016)

The notations and variables of the mixed-integer linear programming model in Chung et al. (2016) are presented as follows:

Notations

n	total number of jobs;
k	the maximum number of batches which is set as n;
B	batch capacity;
r_j	release time of job $j, j = 1, \ldots, n$;
p_{1j}	processing time of job j in the batch machine, $j = 1, \ldots, n$;
p_{2j}	processing time of job j in the single machine, $j = 1, \ldots, n$;

* Corresponding author at: Department of Industrial & Systems Engineering, University of Wisconsin-Madison, Madison, WI 53706, USA.
E-mail address: zhongshun.shi@wisc.edu (Z. Shi).

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s_j size of job $j, j = 1, \ldots, n$;
W limited waiting time of each job in the buffer.

Variables
x_{jb} $x_{jb} = 1$ if job j is assigned to batch b, and otherwise $x_{jb} = 0$;
S_b starting time of batch b in the batch machine, $b = 1, \ldots, k$;
P_b processing time of batch b in the batch machine, $b = 1, \ldots, k$;
C_b completion time of batch b in the single machine, $b = 1, \ldots, k$;
C_{max} the makespan.

Based on the above notations and variables, the mixed-integer linear programming model in Chung et al. (2016), denoted by *CSL* in this paper, is presented as follows:

$$(CSL)\ \ \min C_{max} \tag{1}$$

$$\text{s.t.} \sum_{b=1}^{k} x_{jb} = 1, \quad \forall j = 1, \ldots, n, \tag{2}$$

$$\sum_{j=1}^{n} x_{jb} s_j \leqslant B, \quad \forall b = 1, \ldots, k, \tag{3}$$

$$P_b \geqslant x_{jb} p_{1j}, \quad \forall b = 1, \ldots, k,\ j = 1, \ldots, n, \tag{4}$$

$$S_b \geqslant x_{jb} r_j, \quad \forall b = 1, \ldots, k,\ j = 1, \ldots, n, \tag{5}$$

$$S_b \geqslant S_{b-1} + P_{b-1}, \quad \forall b = 2, \ldots, k, \tag{6}$$

$$C_b \geqslant S_b + P_b + \sum_{j=1}^{n} x_{jb} p_{2j}, \quad \forall b = 1, \ldots, k, \tag{7}$$

$$C_b \geqslant C_{b-1} + \sum_{j=1}^{n} x_{jb} p_{2j}, \quad \forall b = 2, \ldots, k, \tag{8}$$

$$W \geqslant \sum_{j=1}^{n} x_{jb} p_{2j}, \quad \forall b = 1, \ldots, k, \tag{9}$$

$$C_{max} \geqslant C_b, \quad \forall b = 1, \ldots, k, \tag{10}$$

$$x_{jb} \in \{0, 1\}, \quad \forall b = 1, \ldots, k,\ j = 1, \ldots, n. \tag{11}$$

The objective function is to minimize the makespan. The detailed descriptions of constraints (2)–(11) can be referred to Chung et al. (2016). The original paper claimed that constraint (9) can guarantee that the waiting time of each job is limited by an identical W.

A counterexample of 9 jobs whose information is shown in Table 1 to illustrate the problem. Let the batch capacity $B = 3$ and the limited waiting time $W = 6$. We solve model *CSL* with this counterexample by CPLEX 12.6.3 to obtain the optimal solution: first processing batch = (job 1, job 2, job 3), second processing batch = (job 4, job 5, job 6), third processing batch = (job 7, job 8, job 9). The related C_{max} of model *CSL* is 21 for this optimal schedule which is shown in a Gantt chart in Fig. 1.

One error is that constraint (9) cannot guarantee the waiting time of each job is limited by *W*. In Fig. 1, obviously, the waiting time of the last job in the second processing batch is equal to 8 which exceeds $W = 6$.

Another error also exists in constraint (9). The limited waiting time is to restrict the time that elapses between the completion of jobs at stage 1 and the start of processing at stage 2 (Su, 2003), i.e., the waiting time for each job in the buffer should be smaller than or equal to *W*. Therefore, in terms of the last job in one batch, its processing time in the single machine should not be included in the waiting time. Constraint (9) of model *CSL* is so tight that the real optimal solution of this problem will be cut off.

Another counterexample of 3 jobs is given as follows. Let $s_1 = s_2 = s_3 = 1, r_1 = r_2 = r_3 = 0, p_{11} = p_{12} = p_{13} = 100, p_{21} = p_{22} = 2$ and $p_{23} = 4$. The batch capacity *B* is 3 and the limited waiting time *W* is 4. Under the constraint (9), these three jobs cannot be assigned to the same batch. It is easy to check that the optimal objective value of model *CSL* is 204. However, these three jobs can actually be assigned to the same batch. We can let the last job in this batch is the job with the maximum processing time in the single machine. Thus the waiting time of the last job (job 3) is 4, which is feasible. In this situation, the related objective value is 108.

In addition, constraint (9) is used to estimate the feasibility of each obtained solution during the meta-heuristics development in Chung et al. (2016). This leads that the computational results of the proposed meta-heuristics are also not accurate.

Table 1
A counterexample of this problem.

Job	1	2	3	4	5	6	7	8	9
p_{1j}	1	1	1	2	2	2	12	12	12
p_{2j}	2	2	2	2	2	2	2	2	2
r_j	0	0	0	0	0	0	0	0	0
s_j	1	1	1	1	1	1	1	1	1

3. A correct mixed-integer linear programming formulation

In this section, we present a correct mixed-integer linear programming model based on the above analyses. Without loss of generality, we sort the jobs in non-increasing order of p_{2j}. In one batch, the last job should be the job with the maximum processing time of this batch in the single machine, since the waiting time of the last job is smallest in this way. A new variable z_{jb} is introduced. $z_{jb} = 1$ if job j is assigned to batch b and its related processing time p_{2j} is the maximum one among all the jobs assigned to batch b, and otherwise, $z_{jb} = 0$.

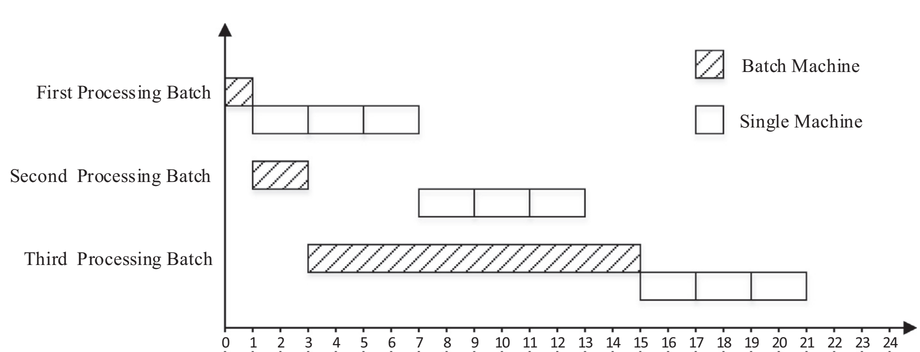

Fig. 1. Gantt chart of the optimal schedule based on *CSL* model.