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A new robust criterion for the vehicle routing problem with uncertain travel time

Lei Wu [a,*], Mhand Hifi [b], Hiba Bederina [c]

[a] Wenlan School of Business at Zhongnan University of Economics and Law, China
[b] Computer Science and Operations Research at the University of Picardie Jules Verne (UPJV), France
[c] UPJV, France

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ABSTRACT

This article investigates a new robust criterion for the vehicle routing problem with uncertainty on the travel time. The objective of the proposed criterion is to find a robust solution which displays better behaviour on a majority of scenarios, where each scenario represents a potential state of an uncertain event. In order to highlight the robustness of the proposed approach, the new robust criterion is compared with the classical robust criteria, such as best-case, worst-case and min-max deviation. Inspired from the mechanism developed by B. Roy for evaluating the robustness, this paper focuses on providing two robust conclusions for the new robust criterion: perfectly robust and pseudo robust. For the perfectly robust, the robust criterion is evaluated by using an exact method on a set of 480 small-scale instances generated from Solomon's benchmark instances. For the pseudo robust, the robust criterion is evaluated by using a metaheuristic on a set of 54 medium-scale and large-scale instances. The numerical results show that the new approach is able to produce the robust solutions in a majority of cases.

1. Introduction

In the past five years, sustained economic growth due to the evolution of information proves that big data technologies are able to constantly improve the efficiency of the social network. Unfortunately, many informations are often unknown or uncertain in real-world applications, such as, the return of risk asset after one year, the travel time for a route in tomorrow morning, etc. Therefore, decision making under uncertainty is encountered in numerous domains such as transportation, logistics, telecommunication, reliability and production management. Despite the technological progress of recent years, tackling some combinatorial optimization problems with uncertainty parameters remains a challenging topic.

Most mathematical models dispose the uncertainty by replacing an uncertain data by a series of deterministic parameters, such as, interval of variation, experience value, variance, etc. For instance, in mathematical programming, some approaches have been already proposed in order to proximately represent uncertain events. From literature, two approaches can be distinguished: (i) sensitivity analysis and (ii) stochastic optimization. The sensitivity analysis can be regarded as a post-occupancy evaluation procedure which aims at finding an interval of variation for each uncertain parameter. Such an interval is used to guarantee the stability of an optimal solution with the consideration that some parameters of the original problem can be perturbed. In stochastic optimization, uncertain events are characterized by their probability distributions. However, the information related to each uncertain event is generally obscure, which complicates the determination of its probability distribution. Therefore, it is worth to try associating each uncertain parameter with a set of values, which can also be labelled as *scenario*. Each scenario corresponds to a potential value that can be reached by an uncertain parameter. In the ideal case, one seeks a solution which displays the best performance over all available scenarios. Such a solution is often difficult to find or, sometimes, does not exist. One of the goals of the robust optimization is to develop decision criteria used to characterize the robustness of the optimization solution. Note that the robust optimization have been proposed for a variety of combinatorial optimization problems (cf., the reader can be referred to Kouvelis & Yu (1997) and Gabrel, Murat, & Thièle (2014)). In this paper, we propose a new robust criterion for the Robust Vehicle Routing Problem (RVRP) with uncertainty on the travel time (cf., Bertsimas & Simchi-Levi, 1996). The proposed approach is based on the theoretical concept developed in Roy (2010): a solution qualified on a majority of scenarios is never too bad.

* Corresponding author at: Wenlan School of Business at Zhongnan University of Economics and Law, 182 Nanhu Avenue, Wuhan 430073, China.
E-mail addresses: wu.lei@zuel.edu.cn, lei.wu@u-picardie.fr (L. Wu).

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Vehicle Routing Problem (VRP), which is a well known NP-hard problem, was first studied by Dantzig and Ramser (1959). An instance of the VRP can be described as follows: given a set of customers and a fleet of vehicles, where all vehicles are characterized by a capacity and each of them is localized at a central depot. The aim of the VRP is to determine the solution supplying all customers with the minimum transport cost. In the standard version of the VRP, the transport cost is usually valued by the travel distance. In real-world applications, the transport cost usually strongly depends on the travel time. Unlike the travel distance, the travel time is often uncertain, especially for its evaluation at some future moment. Recently, in order to develop computational models for handling the uncertainty related to parameters of the problem, it becomes more interesting to design robust models based on the discrete scenarios (cf., Adida & Perakis, 2010; Han, Lee, & Park, 2013).

In this paper, the travel time is represented by a set of scenarios, where each scenario presents a potential value of the travel time required by the vehicle for passing a route. A solution is said to be robust, if it is qualified according to a prefixed robust criterion. In robust optimization models, a robust criterion is often stated as an objective function. Therefore, finding the best solution for a robust criterion is equivalent to determine the best solution optimizing the objective function related to the robust criterion. The robust criteria elaborated in literature are generally based on the preferential unit risk (cf., Zymler, Kuhn, & Rustem, 2013; Zhu & Fukushima, 2009). Among all available robust criteria collected from literature, we cite here: the *best case criterion*, the *worst case criterion* (cf., Solano-Charris, Prins, & Santos, 2015), the *min-max deviation criterion* (cf., Aissi, Bazgan, & Vanderpooten, 2009) and the *bw-robustesse (best-worst) criterion* (cf., Gabrel, Murat, & Wu, 2013).

Besides the determination of models and the development of algorithms, another difficulty for the robust optimization is the evaluation of the solutions (i.e., either optimum or approximate) obtained by using different robust criteria. Roy (2010) proposed three measures of robustness for a considered criterion, which can be summarized as follows.

- ***Perfectly robust conclusions*** : the optimality of the solution has been proven by using an exact algorithm;
- ***Approximately robust conclusions*** : the solution is approximatively determined with providing the approximation ratio;
- ***Pseudo robust conclusions*** : the solution is computed by using metaheuristic without providing any available information on its optimality.

Due to the high complexity of the RVRP, providing the perfectly robust conclusion on a robust criterion becomes impractical when the size of problem increases. Therefore, there are few papers addressing the development of robust criteria for the family of the vehicle routing problem. This paper attempts to propose a mechanism for providing either the perfectly robust conclusion or the pseudo robust conclusion for a considered robust criterion. More precisely, the new robust criterion is evaluated on two sets of benchmark instances. The first set contains the small-scale instances and the second set includes the instances varied from the medium-scale and the large-scale. On the first set, we apply a generic exact solver (i.e., Cplex solver 12.6) to solve the problem to its optimality and we provide the perfectly robust conclusion. The obtained optimum solutions are compared with those provided by using classical robust criteria. On the second set, we adopt a large neighbourhood search-based metaheuristic to approximatively solve the RVRP. In order to highlight the performance of the new robust criterion and the proposed metaheuristic, we compare the obtained approximate solutions with those provided by Cplex solver within a limited runtime.

The remainder of the paper is organized as follows. Section 2 introduces several classical criteria based on an integer linear programming approach for the RVRP. In Section 3, we propose a new criterion for the RVRP and give a numerical example to display its property. Section 4 summarizes the principle of the proposed metaheurisitic. In Section 5, the new criterion is evaluated by using both the exact algorithm and the metaheuristic in order to provide perfectly robust conclusions and pseudo robust conclusions.

2. Model with discrete set of scenarios

In this section, we describe some definitions and notations which will be used in the rest of paper. Given a central depot (noted by v_0), a set of n customers $V = \{1,\ldots,n\}$ and a set of identical vehicles, each customer i has a quantity c_i of goods to be delivered and each vehicle is characterized by a capacity C. The goal of the VRP is to determine a list of feasible routings serving all customers with a *minimum travel distance* and using a *minimum number vehicles*. Therefore, an instance of the VRP can be represented as a complete graph $G = (V,E)$, where V denotes the set of vertices and $E = \{(i,j) \mid i,j \in V\}$ the set of edges. Therefore, a standard linear program for the VRP (cf., Toth & Vigo, 2002) can be written as follows:

$$(\text{ILP}_{vrp}) \quad \min \ \mu \times m + \sum_{i=1}^{n}(t_{0i}x_{0i} + t_{i0}x_{i0}) + \sum_{i=1}^{n}\sum_{j=1}^{n} t_{ij}x_{ij} \tag{1}$$

$$\text{s.c.} \quad \sum_{i=1}^{n} x_{0i} = \sum_{i=1}^{n} x_{i0} \leqslant m, \tag{2}$$

$$\sum_{i=0}^{n} x_{ik} = 1, \qquad \forall\, k = 1,\ldots,n, \tag{3}$$

$$\sum_{j=0}^{n} x_{kj} = 1, \qquad \forall\, k = 1,\ldots,n, \tag{4}$$

$$l_j - l_i + C(1 - x_{ij}) \geqslant c_j, \qquad \forall\, i \neq j = 1,\ldots,n, \tag{5}$$

$$c_i \leqslant l_i \leqslant C, \qquad \forall\, i = 1,\ldots n, \tag{6}$$

$$m \in \mathbb{N},\ l_i \in \mathbb{N},\ \forall\, i = 0,\ldots,n,\ x_{ij} \in \{0,1\},\ \forall\, i \neq j = 0,\ldots,n, \tag{7}$$

where, t_{ij} presents the travel time from customer i to j; l_i is the current load of the vehicle when customer i is served; the decision variable $x_{ij} = 1$ if edge (i,j) is chosen in the route, 0 otherwise; m presents the number of used vehicles. The objective function (1) aims at minimizing simultaneously the number of used vehicles and the total travel time. The parameter μ measures the impacts of minimizing the number of used vehicles in the objective function. Inequality (2) ensures that the number of vehicles which depart from (or return to) the depot cannot exceed m. Constraints (3) and (4) are used to ensure the property of the cycle, where a cycle departs from and returns to the same depot. As a single route is represented by a single cycle, constraints (5) are used to eliminate all subtours. Constraint (6) is used to ensure the capacity of constraint of the vehicle. Finally, constraint (7) guarantees the integrality of all decision variables.

In literature, constraints (5) are referenced as Miller-Tucker-Zemlin (MTZ) subtour elimination constraints, which are usually considered as weak subtour elimination constraints (cf., Miller, Tucker, & Zemlin, 1960). In contrast to MTZ, Dantzig-Fulkerson-Johnson (DFJ) subtour elimination constraints are known by the community as strong subtour elimination constraints (cf., Dantzig, Fulkerson, & Johnson, 1954). DFJ constraints are based