Computers & Industrial Engineering 112 (2017) 57–70

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Phase II monitoring and diagnosis of autocorrelated simple linear profiles

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ARTICLE INFO

Article history:
Received 4 November 2016
Received in revised form 22 July 2017
Accepted 4 August 2017
Available online 5 August 2017

Keywords:
Simple linear profiles
Change-point estimation
Autocorrelation
Exponentially weighted moving average

ABSTRACT

If the quality of a process is better represented by a functional relationship between response variables and explanatory variables, a collection of this type of quality data is called a profile. In this paper, we consider the functional relationship which can be represented by a simple linear regression model with a first-order autocorrelation between error terms. We propose exponentially weighted moving average (EWMA) charting schemes to monitor this type of profile. The simulation study shows that our proposed methods outperform the existing schemes based on the average run length (ARL) criterion. We also propose a maximum generalized likelihood ratio method to obtain a change-point estimator to help users determine the assignable causes.

1. Introduction

Control charts, the most successful statistical process control tools, have been successfully applied in various industries and other areas. If the quality of a process can be adequately represented by the distribution of one quality characteristic or of multivariate quality characteristics, control charts are often used to monitor the parameters of the distributions. However, for an increasing number of practical applications, the quality of a process or product may be better represented by a functional relationship between response variables and explanatory variables, also called a profile. When the profile changes, the quality of the process is out of control. Therefore, control charts are used to monitor the stability of this functional relationship (profile) over time. Profile monitoring in statistical quality control has experienced rapid development in the last 10 years. Extensive discussions on profile monitoring have been provided by Woodall, Spitzner, Montgomery, and Gupta (2004), Woodall (2007), Noorossana, Saghaei, and Amiri (2011) and Montgomery (2012, chap. 10).

Profiles may be classified into linear, generalized linear, and nonlinear profile models according to their functional relationships. Many studies have focused on linear profiles. Kang and Albin (2000) and Kim, Mahmoud, and Woodall (2003) proposed different types of charting schemes to monitor simple linear profiles. Mahmoud and Woodall (2004) proposed a global F test method to monitor multiple linear profiles in Phase I. Zou, Tsung, and Wang (2007) recommended a multivariate EWMA (MEWMA) charting scheme to monitor all the parameters of general linear profiles in one chart. For monitoring multivariate simple linear profiles, Noorossana, Eyvazian, and Vaghefi (2010) proposed three types of charting schemes in Phase II. Wang and Wang (2016) studied simple linear profiles with a mixture of normally distributed error terms. More studies on the monitoring of linear profiles can be found in the literature—see, for example, Zou, Zhou, Wang, and Tsung (2007), Li and Wang (2010), Noorossana, Vaghefi, and Dorri (2011), Mahmoud (2012), and Ghahyazi, Niaki, and Soleimani (2014).

Although the monitoring of linear profiles is important, in some practical applications the quality data cannot be adequately represented by linear profile models. For example, the dose-response profile (see Williams, Woodall, & Birch (2007)) and white wine profile (see Huwang, Wang, Yeh, & Huang (2016)) are represented by nonlinear and generalized linear models, respectively. Hence, an increasing number of research has focused on nonlinear and generalized linear profile monitoring, including that of Ding, Zeng, and Zhou (2006), Jensen and Birch (2009), Shiau, Huang, Lin, and Tsai (2009), Yeh, Huwang, and Li (2009), Vaghefi, Tajbakhsh, and Noorossana (2009), Qiu and Zou (2010), Chuang, Hung, Tsai, and Yang (2013) and Amiri, Koosha, Azhdari, and Wang (2015).

The aforementioned studies have assumed that the profile data are independent. However, in many applications, the profile data show correlations over time. For example, the auto paint data could be represented by correlated linear models because the paint thickness (dependent variable) changes depending on the body location (explanatory variable) and the previous paint thickness will affect the next paint thickness. See Noorossana, Amiri, and

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http://dx.doi.org/10.1016/j.cie.2017.08.006
0360-8352/

Soleimani (2008) for more details. Correlated profile data can be found not only in industry but also other areas. Shumway, Azari, and Pawitan (1988) studied mortality fluctuations in Los Angeles County. They found there is a linear relationship between mortality and pollutant particulates and temperature. Furthermore, the residuals of the linear model show that there is an autocorrelation between the error terms (see Shumway & Stoffer (2011)). It is of interesting in monitoring the stability of this relationship over time.

Therefore, there are more and more studies being done on correlated profile monitoring. Soleimani and Noorossana (2012) investigated the effect of three types of within-profile correlation assumptions, AR(1), MA(1), and ARMA(1,1), in monitoring multivariate linear profiles in Phase II. Simulation results indicated that the correlations significantly affected the average run length (ARL) performance of the three existing common methods. Hence, it is necessary to propose a new monitoring scheme when profile data are not independent. Studies of monitoring within-profile correlation models include those of Soleimani, Noorossana, and Amiri (2009), Soleimani, Noorossana, and Narvand (2011) and Keramatpour, Niaki, Khedmati, and Soleymanian (2013). Some studies are focus on monitoring between-profile correlation models. Noorossana et al. (2008) proposed three types of control schemes based on residuals to monitor a simple linear profile with a first-order autocorrelation between error terms. Kazemzadeh, Noorossana, and Amiri (2010) proposed T^2 and EWMA/R charts to monitor autocorrelated polynomial profiles in Phase II. Soleimani and Noorossana (2014) investigated the Phase II monitoring of multivariate simple linear profiles when the independence assumption between profiles was violated. Khedmati and Niaki (2016) proposed a T^2 chart for monitoring the general linear profile with between-profile autocorrelations.

In the present paper, we study the Phase II monitoring for a simple linear regression model with a first-order autocorrelation between the error terms (hereinafter referred to as autocorrelated simple linear profiles). We develop EWMA-type charting schemes that are more effective than existing schemes for monitoring autocorrelated simple linear profiles. Furthermore, a change-point estimator based on maximum likelihood ratio approach for autocorrelated simple linear profiles is proposed to identify when the process has changed after an out-of-control signal has appeared. The remainder of this paper is organized as follows. Existing important monitoring schemes are reviewed in Section 2. Our proposed monitoring schemes are presented in details in Section 3. The simulation study for comparing our proposed schemes with existing monitoring schemes is given in Section 4. Section 5 provides a change-point estimator of a sustained shift in the parameters. In Section 6, an real example is used to illustrate the implementation of the proposed methodology. Conclusions are given in the final section.

2. Existing Phase II methods

In this paper, we focus on the study of a Phase II monitoring for autocorrelated simple linear profiles. Specifically, assume that at a given time j, we have the collected sample $\{(x_i, y_{ij})|\ i = 1,2,\ldots,n\}$, where y_{ij} is the response variable and x_i is the explanatory variable fixed for all profiles. The underlying in-control model is

$$\begin{aligned} y_{ij} &= A_0 + A_1 x_i + \varepsilon_{ij}, \\ \varepsilon_{ij} &= \phi\varepsilon_{i(j-1)} + a_{ij}, \quad i = 1,2,\ldots,n, \quad j = 1,2,\ldots, \end{aligned} \tag{1}$$

where ε_{ij} denotes the correlated error terms, ϕ is the autocorrelation coefficient, and $a_{ij} \overset{\text{i.i.d.}}{\sim} N(0,\sigma^2)$. Furthermore, A_0, A_1, σ^2, and ϕ are assumed known during Phase II. In this study, we assume ϕ is a constant, not a parameter. Therefore, for profile data in model (1), there are three parameters, A_0, A_1, and σ^2, that must be monitored. There have been four studies about monitoring linear profiles with first-order autocorrelation between the error terms, namely Noorossana et al. (2008), Kazemzadeh et al. (2010), Soleimani and Noorossana (2014) and Khedmati and Niaki (2016). Noorossana et al. (2008) proposed three types of control schemes based on residuals to monitor the same profile data as model (1): EWMA/R, T^2, and three EWMA control charts. According to their simulation results, the most effective scheme among the three schemes is the three EWMA control charts (EWMA3 hereinafter) based on the ARL criterion. Kazemzadeh et al. (2010) and Soleimani and Noorossana (2014) used the same monitoring statistics and charting schemes as Noorossana et al. (2008) to monitor polynomial profiles and multivariate simple linear profiles, respectively. Therefore, when the polynomial profile and multivariate simple linear models are reduced to the simple linear profile model, the schemes proposed by Kazemzadeh et al. (2010), Soleimani and Noorossana (2014) are exactly the same as the schemes proposed by Noorossana et al. (2008). Khedmati and Niaki (2016) proposed a T^2 chart with a U statistic to monitor general linear profiles; consequently, the control scheme can be applied to model (1). In their simulation study, the T^2 control chart with the U statistic performed slightly better than the T^2 control scheme proposed by Noorossana et al. (2008) for detecting intercept and slope shifts. However, when compared with the EWMA3 scheme, the EWMA3 scheme outperformed the T^2 control chart with the U statistic for detecting intercept, slope, and variance shifts. Thus, we introduce only the most effective existing scheme, the EWMA3 scheme, in this section.

The EWMA3 scheme proposed by Noorossana et al. (2008) is a modification of the EWMA3 approach proposed by Kim et al. (2003) for monitoring the change of the intercept, slope, and variance of the profile data in model (1). Just as Kim et al. (2003) did in their study, Noorossana et al. (2008) first centered controllable values to obtain the following alternative form of model (1) by:

$$\begin{aligned} y_{ij} &= B_0 + B_1 x_i^* + \varepsilon_{ij}, \\ \varepsilon_{ij} &= \phi\varepsilon_{i(j-1)} + a_{ij}, \quad i = 1,2,\ldots,n, \quad j = 1,2,\ldots, \end{aligned} \tag{2}$$

where $x_i^* = x_i - \bar{x}$, $B_0 = A_0 + A_1\bar{x}$, and $B_1 = A_1$. The least squares estimators of B_0 and B_1 for the jth profile in model (2) are $\hat{B}_{0j} = \sum_{i=1}^{n} y_{ij}/n$ and $\hat{B}_{1j} = \sum_{i=1}^{n} x_i^* y_{ij}/S_{xx}$, where $S_{xx} = \sum_{i=1}^{n} x_i^{*2}$. Note that because the controllable values are centered, $\hat{B}_{0j}$ and $\hat{B}_{1j}$ are independent. Because of the AR(1) structure between the error terms, the jth and $j-1$th estimators for B_0 and B_1 in model (2) follow the relationship

$$\begin{aligned} \hat{B}_{0j} &= \phi\hat{B}_{0(j-1)} + (1-\phi)B_0 + r_j \quad \text{and} \\ \hat{B}_{1j} &= \phi\hat{B}_{1(j-1)} + (1-\phi)B_1 + u_j, \end{aligned}$$

where $r_j \overset{\text{i.i.d.}}{\sim} N(0,\sigma^2/n)$ and $u_j \overset{\text{i.i.d.}}{\sim} N(0,\sigma^2/S_{xx})$. Noorossana et al. (2008) then calculated the residuals of the intercept and slope models as follows:

$$\begin{aligned} e_{Ij} &= \hat{B}_{0j} - \phi\hat{B}_{0(j-1)} - (1-\phi)B_0 \quad \text{and} \\ e_{Sj} &= \hat{B}_{1j} - \phi\hat{B}_{1(j-1)} - (1-\phi)B_1, \end{aligned} \tag{3}$$

where $e_{Ij} \overset{\text{i.i.d.}}{\sim} N(0,\sigma^2/n)$ and $e_{Sj} \overset{\text{i.i.d.}}{\sim} N(0,\sigma^2/S_{xx})$. Finally, Noorossana et al. (2008) used e_{Ij} and e_{Sj} to construct the EWMA statistics for monitoring the intercept and slope change.

For monitoring the intercept, the EWMA statistic is

$$\text{EWMA}_I(j) = \theta e_{Ij} + (1-\theta)\text{EWMA}_I(j-1),$$

where $\text{EWMA}_I(0) = 0$ and θ $(0 < \theta \le 1)$ is a smoothing constant. Because e_{Ij} follows an independent and identical normal distribu-