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Improving the evaluation of cross efficiencies: A method based on Shannon entropy weight

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ABSTRACT

Data envelopment analysis (DEA) is a non-parametric statistical method used to assess the production frontiers of decision-making units (DMUs) and evaluate their relative efficiencies. However, using traditional DEA models to evaluate efficiency has certain deficiencies. For example, some DMUs cannot be ranked fully using traditional DEA models. To solve such problems, the cross-efficiency evaluation method has been proposed to replace the self-evaluation system. Nevertheless, this method, which uses a mutual evaluation system to overcome the ranking issue, still has shortcomings such as non-unique cross efficiency weights, which may result in multiple cross efficiency values. Further, providing adequate performance improvement tools to decision makers is difficult using only the average efficiency values. To address the problems of uniqueness and aggregation, this study proposes two cross efficiency models, designated MAX and MIN models. The self-evaluated optimal weight of a certain DMU derived from these MAX and MIN models can maximize or minimize the efficiency of the DMU to form two cross efficiency matrices, which can partially solve the problem that results from multiple optimal weights. To solve the aggregation problem of cross efficiency, the study also applies Shannon entropy, which classifies all cross efficiency values into one group of acquired common objective weights to avoid subjective factors. Finally, the present study confirms an improvement when using the proposed method by examining production data on 15 thermoelectric enterprises in China.

1. Introduction

Data envelopment analysis (DEA), originally presented by Charnes, Cooper, and Rhodes (CCR) (1978), is a non-parametric statistical method for assessing the production frontier of decision-making units (DMUs) and evaluating their relative efficiencies. Over the past 35 years, a large number of theoretical innovations based on traditional CCR DEA methods have emerged. Banker, Charnes, and Cooper (1984), for example, proposed the BCC model to analyze efficiencies based on variable returns to scale, while Tone's (2001) slack-based model can be used to consider input and output slacks. In addition, several fuzzy DEA methods have been proposed to solve specific problems (Hatami-Marbini, Saati, & Tavana, 2010). These models have been widely applied in both profit and non-profit organizations such as banks (Avkiran, 2015), educational institutions (Johnes, 2006), hospitals (Biørn, Hagen, Iversen, & Magnussen, 2003), and others (Liang, Yang, Cook, & Zhu, 2006; Wu, Chu, Sun, Zhu, & Liang, 2016; Wu, Zhu, Chu, Liu, & Liang, 2015).

However, both traditional DEA models and their extensions have many disadvantages in sequencing efficiency values. For example, traditional DEA models cannot rank all DMUs fully, especially the more efficient DMUs (Wang & Chin, 2010). In addition, they typically prefer self-evaluated weights to evaluated units, meaning that any advantages may be overstated and the disadvantages overlooked (Liang, Wu, Cook, & Zhu, 2008a). To solve these problems, scholars have started to improve traditional DEA models. The cross efficiency evaluation (CEE) method (Liang et al., 2008a; Sexton, Silkman, & Hogan, 1986; Wang & Chin, 2011) is one typical such development, which aims to overcome the disadvantages of relying on self-evaluation by using mutual evaluation. In particular, in CEE, each DMU will have a self-evaluated efficiency using its own optimal weights, while the remaining DMUs' peer-evaluated efficiencies use the optimal sets of weights of these remaining DMUs. Then, all the efficiencies for each DMU are averaged into an efficiency value to get its cross-efficiency score. A mutual evaluation system enables the sequencing of all DMUs

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from the perspective of the global optimum, thereby removing extreme and unrealistic weights and leading to its wide application by previous authors (Falagario, Sciancalepore, Costantino, & Pietroforte, 2012; Gutierrez & Ruiz, 2013; Lim, Oh, & Zhu, 2014).

Although the CEE method has been widely applied because of its many advantages, it still has certain drawbacks. One of these is the problem of non-uniqueness of optimal weights for each DMU (Sexton et al., 1986). Since the optimal weight of cross efficiency is not usually unique, its generation is still arbitrary, and different calculation routines may thus obtain different cross efficiency values (Despotis, 2002). In addition, an aggregated cross efficiency value based on the average is not Pareto-optimal, which makes it difficult to provide improved methods to decision makers, and there is no relevant relation between efficiency value and weight. Based on these shortcomings, the present study extends the CEE method from the aspects of both aggregation and uniqueness.

On the aspect of cross efficiency weights and data characteristics, Stewart (1996) proposed randomly observing the cross efficiency probability distribution after setting known probability distributions for input and output. Similarly, Salo and Punkka (2011) proposed a ranking intervals reflecting the best and worst efficiency rankings, while Chen and Zhu (2011) assessed their changes by using efficiency bootstraps. They assumed that input and output weights are random variables and that the efficiency of DMUs is also derived randomly. Du, Cook, Liang, and Zhu (2014) considered that enterprises compete and cooperate, and used a cross efficiency matrix to analyze cost and resource allocation problems. In addition, Lim et al. (2014) evaluated investment options in the Korean stock market using the CEE method and constructed the mean value variance expression of these investment portfolios.

In order to solve the non-uniqueness of the traditional CEE method, Sexton et al. (1986) and Doyle and Green (1994) introduced different secondary objectives and formulated a secondary DEA model that can be divided into two strategies: benevolent and aggressive. A benevolent strategy aims to maximize the efficiencies of other DMUs, under the condition that the self-evaluated efficiency value of the evaluated unit is unchanged; while an aggressive strategy aims to minimize the efficiencies of other DMUs, under the same condition. Liang, Wu, Cook, and Zhu (2008b) further extended these two strategies by introducing different secondary objective functions. Each new secondary function represents different efficiency evaluation criteria and can thus be applied to different practical circumstances. Wang and Chin (2010) further pointed out that each standard efficiency value in Liang et al.'s (2008a) model is 1, which ineffective DMUs cannot achieve. Therefore, these authors improved the model by taking the CCR efficiency value of each unit as the new standard value. Similar ideas were also put forward by Lim (2012), who introduced minimax and maximin functions into the cross efficiency secondary objective function. Recently, Wu et al. (2016) proposed several secondary goal models to select weights considering both desirable and undesirable cross efficiency targets of all the DMUs.

Nevertheless, scholars such as Wu, Sun, and Liang (2012) and Wang, Chin, and Wang (2012) have recently pointed out that no models (benevolent, aggressive, or neutral) consider the phenomenon of zero weight. When the weight is zero, it means that the relevant input or output has played no role in the efficiency evaluation, which is clearly unreasonable. Therefore, they constructed a new weight-balanced model in which each input and output variable plays as similar a role as possible in the efficiency evaluation.

Although studies of cross efficiency have tended to focus on the problem of uniqueness and overlook the aggregation of cross efficiency, some refinements have been proposed in this regard. For instance, Wu, Sun, and Liang (2011) introduced information entropy and established a new cross efficiency model using Shannon entropy, while Zerafat Angiz, Mustafa, and Kamali (2013) proposed a cross ranking of DMUs. According to this method, a cross efficiency matrix is first transformed into a ranking sequence matrix. Then, the weight of the cross efficiency aggregation is calculated by using the sequence priority model and a weighted ranking is carried out. However, Yang, Yang, Liu, and Li (2013) indicated that the preferences of decision makers should also be considered when aggregating cross efficiency values. Hence, they combined evidence reduction with the CEE method, first producing a cross efficiency matrix and then aggregating using the former approach.

In summary, these traditional secondary goal methods based on optimal weights of a certain DMU have two main deficiencies. First, the traditional secondary goal model just considers only one perspective by maximizing or minimizing other DMUs' efficiencies while maintaining a certain DMU's optimal efficiency. Second, the effects of the efficiency values of the same DMU under the self-evaluated optimal weights of other DMUs may differ compared with the final efficiency value of this DMU, while the traditional CEE method applies the same weights to these cross efficiency values. Hence, to improve the efficiency evaluation results, it is necessary to consider the cross efficiency matrices according to different weights.

To overcome these issues, this paper considers the perspectives obtained by both maximizing and minimizing other DMUs' efficiencies; that is, MAX and MIN models are put forward in this study. These models aim to maximize and minimize the efficiency values of other DMUs under the self-evaluation condition of a certain DMU to acquire a series of efficiency vectors and then calculate the cross efficiency matrix by repeating the above steps for all DMUs. Further, the possible multiple optimal weights of DMUs under self-evaluation conditions are fully considered for both cross efficiency matrices. This method can reduce the impact of multiple optimal weights and considers possibilities synthetically when evaluating cross efficiency matrices, because the matrix derived from the MAX model benefits DMUs, while that from the MIN model is detrimental to them. The final cross efficiency matrix can thus be obtained by calculating the geometric mean values of the relevant items of these two matrices. Hence, because this final matrix integrates the MAX and MIN models, it can solve the non-uniqueness of efficiency that results from using multiple optimal weights.

As for the aggregation problem, Shannon entropy is applied in this study. As noted earlier, traditional cross efficiency matrices apply the same weights to cross efficiency values; however, different cross efficiency values have different impacts on the final efficiency values of DMUs. By contrast, information entropy theory takes the cross efficiency values of DMUs as expressions of their final efficiency values under different optimal weights, which can then be integrated into their final efficiency values. Shannon first introduced information entropy in his paper A Mathematical Theory of Communication (Shannon, 1948). According to his view, the quantity and quality of information is a major determinant of the accuracy and reliability of decision-making systems. Information entropy is adopted to measure the expected value of a random variable; the greater the entropy of a variable, the more situations in which it appears. We then need more information to determine this variable. Information entropy is therefore a good indicator in making a wide range of evaluations. In this study, we propose an entropy model to obtain a set of weights for aggregating the cross efficiency, instead of traditional average cross efficiency. Compared with the subjective assignment of weights, Shannon entropy can thus apply more objective weights to the cross efficiency matrix.

The structure of this paper is as follows. The traditional cross efficiency model is given in Section 2. In Sections 3 and 4, the