



# Reliability improvement through designed experiments with random effects



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## ABSTRACT

Design of experiment (DOE) is a useful tool to identify significant factors and choose factor levels for product reliability improvement. In practice, practitioners often ignore random effects that result from the experimental protocol in the reliability experiment. In this paper, we consider product reliability improvement with designed experiment when the test is actually not completely randomized. The Weibull distribution is used to model the lifetime, leading to a smallest extreme value distribution for the log-lifetime. Random effects are incorporated into the model through mean time to failure (MTTF). We improve the product reliability with maximizing the MTTF. The simulation study shows that ignoring random effects in modeling can result in unreliable factors identification and estimation. We also illustrate the proposed method with a real example.

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## 1. Introduction

Because of the fierce competition in global market, and the increasing demands of customers, industrial engineers must strive to improve product quality. Design of experiment (DOE) is an important tool for product quality improvement. It has received a great deal of attention of practitioners and researchers. Much of the pioneering work was performed by R. A. Fisher at the Rothamsted Agricultural Experimental Station in the 1930s. And since then, DOE has been successfully and widely used for product and process improvement (Antony, 2000; Artiles-León & Mella-Cabrera, 1997; Bursali, Ertunc, & Akay, 2006; Ding, Xu, Hopper, Yang, & Ho, 2013; Fowler & Rogers, 2015; Lewis, Montgomery, & Myers, 2001; Montgomery, 1992; Myers, Montgomery, & Anderson-Cook, 2016; Wu & Hamada, 2009).

Condra (2001) described the reliability as quality over time. Reliability improvement is an important part of product quality enhancement. The DOE technique is extensively used in quality assessment and improvement experiments. However, reliability improvement experiments are more difficult to conduct than quality improvement experiments (Joseph & Yu, 2006). This is mainly due to two aspects. One is that the data are usually non-normal. The lifetime data usually follow an Exponential, Weibull, Lognor-

mal, or Gamma distribution. The other aspect is that the lifetime data are usually censored. Because of the time, cost and other constraints, engineers often stop the experiment at a predetermined time or a predefined number of failures. Thus, some test units may not fail at the end of the test and the failure time is not known exactly. The presence of censoring makes the data contain less information than the uncensored data and are more difficult to analyze. Thus, it is challenging to apply the DOE tool for reliability improvement.

In conducting a DOE to improve product reliability, engineers generally consider two goals: (a) identifying the significant factors affecting the lifetime and (b) choosing the factor levels which lead to reliability improvement. Condra (2001) provided several examples from electronic industry of using DOE to improve product reliability. Bullington, Lovin, Miller, and Woodall (1993) illustrated a detailed description of designed experiment to improve the lifetime of a thermostat. Hamada (1993) demonstrated the usefulness of Taguchi's robust design for improving the lifetime of drill bits. Hamada (1995) reported the DOE tool for reliability improvement, and they presented several examples in their paper. Van den Bogaard, Shreeram, and Brombacher (2003) discussed the improvement of product reliability through maximizing mean time to failure (MTTF). Joseph and Yu (2006) emphasized the importance of DOE in quality and reliability improvement. Besseris (2010) presented a case study from aluminum milling operations to illustrate the reliability enhancement through DOE, with the

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aim of maximizing the MTTF. Response surface designs are also widely used in reliability improvement experiments for determining the optimal designs (Das, Kim, & Park, 2015; Das & Lin, 2011; Wang & Hsu, 2009). The aforementioned literatures clearly demonstrate the importance of DOE technique for reliability improvement. In this paper, we propose a new model for modeling the effects of experimental factors on lifetime. The improvement of product reliability is obtained through choosing the critical factors and factor levels with the aim of maximizing MTTF.

Randomization is one of the fundamental principles of DOE. In many industrial experiments, complete randomization design is impossible. There are different batches in one experiment, and the lifetime of units in one batch may be heteroscedastic and correlated. Also there may be natural or artificial cluster that the lifetime of subjects within the same cluster are correlated. Kensler, Freeman, and Vining (2014) noted that the reliability experiments are not completely randomized under the present cost and time constraints. Random effects may exist due to the experimental protocol in reliability experiments, such as, subsamples, blocks, cluster structures, split-plot structures, and grouped data. It is important to incorporate an unobserved random effect into the lifetime model to account for the effect of experimental protocol. Feiveson and Kulkarni (2000) illustrated the importance of random batch effects associated with spools of fiber in reliability data analysis. Leon, Li, Guess, and Sawhney (2009) showed that it may lead to seriously misleading results without considering the batch effects. Freeman and Vining (2010, 2013) presented the analysis of reliability data with subsampling, and they concluded that ignoring random effects may overestimate the significance of factors and underestimate the experimental error. Wang, Niu, and He (2015) illustrated the reliability analysis of grouped data. Lv, He, and Vining (2017) proposed the two stage method to incorporate the random effects into the models of quality and reliability characteristics. A large number of researchers also noted the importance of incorporating random effects into the analysis to reflect the experimental designed protocol (Kensler, Freeman, & Vining, 2015; Kensler et al., 2014; Lv, Niu, Qu, He, & He, 2015; Seo & Pan, 2016; Xiao & Tang, 2013). All previous work stressed the importance of random effects in the analysis of reliability data. However, they did not focused on the reliability improvement. As we mentioned before, DOE is an important tool for product reliability improvement. Thus, in this paper, we mainly focus on product reliability improvement via statistically designed experiments when the experiments are constrained randomization.

In this paper, we compare two models in the reliability improvement. The traditional model assumes the experiments are completely randomized and there are no random effects in the model. The proposed model assumes that random effects are incorporated into the model through MTTF. The rest of this paper is organized as follows. Section 2 introduces the Weibull regression model with the random effects. Section 3 suggests the method for estimating the parameters. Section 4 presents a simulation study to illustrate the advantage of the random effects model. Section 5 gives an application of the proposed method on a real dataset. Section 6 presents the conclusions and further directions.

## 2. Methodology

The location-scale and log-location-scale models are commonly used to model the lifetime data. The Weibull distribution is one of the most frequently used members of the log-location-scale models. Because of its flexibility in modeling different failure rate, it has been commonly used in different conditions. In this paper, we assume a Weibull distribution for the lifetime data. It is convenient to take a logarithm of the lifetime  $T$ , and perform inference on the

location-scale model. A common way to express the Weibull and SEV model is proposed by Hamada (1993, 1995):

$$Y = \log(T) = X\theta + \sigma\varepsilon, \quad (1)$$

where  $X$  are the factor effects,  $\theta$  are the factor effect estimates,  $\sigma$  is the scale parameter, and  $\varepsilon$  represents the standard SEV value.

If the lifetime  $T$  has a Weibull distribution,  $T \sim \text{Weibull}(\eta, \beta)$ , then  $Y = \log(T)$  follows a smallest extreme value (SEV) distribution. The probability density function (pdf) and the corresponding cumulative distribution function (cdf) of the SEV distribution are

$$\begin{aligned} f(y; \mu, \sigma) &= 1/\sigma \exp\left(\frac{y-\mu}{\sigma} - \exp\left(\frac{y-\mu}{\sigma}\right)\right), \\ F(y; \mu, \sigma) &= 1 - \exp\left(-\exp\left(\frac{y-\mu}{\sigma}\right)\right), \end{aligned} \quad (2)$$

where  $\sigma = 1/\beta$  is the scale parameter and  $\mu = \log(\eta)$  is the location parameter.

The mean of the SEV distribution is  $E(Y) = \text{MTTF} = \mu - \sigma\gamma$ ,  $\gamma \approx 0.5772$  is the Euler's constant (Meeker & Escobar, 1998). Another way to express the SEV distribution is

$$\begin{aligned} f(y; \text{MTTF}, \sigma) &= 1/\sigma \exp\left(\frac{y-\text{MTTF}-\sigma\gamma}{\sigma} - \exp\left(\frac{y-\text{MTTF}-\sigma\gamma}{\sigma}\right)\right), \\ F(y; \text{MTTF}, \sigma) &= 1 - \exp\left(-\exp\left(\frac{y-\text{MTTF}-\sigma\gamma}{\sigma}\right)\right). \end{aligned} \quad (3)$$

In order to improve product reliability, engineers try to find out the factors that maximize eventually the mean lifetime. The recommended levels of the significant factors are those levels which maximize MTTF. The MTTF is important in product reliability analysis and improvement. As noted by Borgonovo, Aliee, Glaß, and Teich (2016), the MTTF is an intuitive concept and widely used by practitioners, they proposed a new reliability importance measure based on the change in MTTF to determine the importance of the components in the system. Das et al. (2015) used the designed experiments to improve the mean lifetime of a system or engineering process. Han (2015) also assumed that the relationship between the mean lifetime and stresses is log-linear under the exponential accelerated failure time model. For the SEV distribution, MTTF depends linearly on the location and scale parameters. Eq. (1) shows that the factors have a linear relationship on the mean  $\log(\text{lifetime})$ . In this paper, we assume both fixed factor effects and random effects influence the mean  $\log(\text{lifetime})$ . They enter into the model through the linear relationship with MTTF of  $\log(\text{lifetime})$ . Then, we can express the relationship between MTTF and experimental factors and random effects as follows

$$E(Y)_i = \text{MTTF}_i = \mathbf{x}_i^T \boldsymbol{\gamma} + u_i, \quad (4)$$

where  $\mathbf{x}_i$  is the vector of factor levels in the  $i$ th test stand,  $\boldsymbol{\gamma}$  is the vector of the fixed factor effects coefficients,  $u_i$  is the random effect which follows a normal distribution with  $u_i \sim \text{Normal}(0, \xi^2)$ .

Another common assumption in designed experiments for reliability test is that the failure mechanism is the same among different test stands. This means that the shape parameter of Weibull distribution is constant under different conditions. Thus, the scale parameter ( $\sigma$ ) of SEV distribution is assumed to be constant.

## 3. Parameter estimation and inference

Due to the property of reliability data, the least square method commonly used in designed experiments analysis cannot be directly used to analyze lifetime data. Most statisticians use Maximum Likelihood Estimation (MLE) because of its simplicity to handle censored data. It has been widely used in reliability data analysis (Lawless, 2003; Meeker & Escobar, 1998). Here, we derive the maximum likelihood estimates of model parameters.

Three types of censored data are left censored, right censored and interval censored. This paper mainly focuses on right censored

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