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A 0-1 mixed programming model based method for group decision making with intuitionistic fuzzy preference relations

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ABSTRACT

Intuitionistic fuzzy preference relations (IFPRs) are important and powerful that can express the decision makers' preferred and non-preferred judgements simultaneously. To rank objects reasonably, consistency analysis is necessary. Thus, this paper defines a new multiplicative consistency concept for IFPRs that is a natural extension of the crisp case. Using the new concept, 0–1 mixed programming models are constructed to judge the consistency of IFPRs and to determine missing values in incomplete IFPRs. Considering the inconsistent case, an approach to deriving multiplicative consistent IFPRs is presented. To address group decision making with IFPRs, a consensus index is proposed to measure the agreement degree between individual IFPRs. Then, an approach to group decision making with IFPRs is presented that can address the inconsistent and incomplete IFPRs. Finally, two practical examples are provided to show the efficiency and feasibility of the new theoretical results, and a comparison analysis is performed.

1. Introduction

In practical decision-making problems, it often needs the decision makers (DMs) to evaluate and to rank a finite set of objects. In this process, preference relations are one of the most commonly used techniques, which have been received considerable attentions both in theory and application. According to the construction of elements in preference relations, they can be classified into three types: multiplicative preference relations (Saaty, 1980), reciprocal preference relations (Tanino, 1984), and linguistic fuzzy preference relations (Herrera, Herrera-Viedma, & Verdegay, 1996). Due to the complexity of the decision-making problems, it becomes more and more difficult to require the DMs to give the exact judgments. To address this problem, researchers turned to Zadeh's fuzzy set theory, and proposed preference relations with fuzzy numbers (Buckley, 1985; Saaty & Vargas, 1987; van Laarhoven & Pedrycz, 1983; Xu, 2001, 2002).

Although preference relations with Zadeh's fuzzy sets can well express the DMs' vagueness, they give the preferred degree of an object over another. Sometimes, it is insufficient to fully address the DMs' opinions because the DMs might also provide the non-preferred degree between a pair of compared objects. To address this issue, Atanassov's intuitionistic fuzzy sets (Atanassov, 1986) are good choices, which can denote the DMs' preferred information, non-preferred information and hesitant information simultaneously. Xu (2007a) first noted the advantages of Atanassov's intuitionistic fuzzy sets and introduced intuitionistic fuzzy preference relations (IFPRs), which can be seen as an extension of reciprocal preference relations. After the original work of Xu (2007a), the theory and application of IFPRs have been largely developed in last ten years (Behret, 2014; Gong, Li, Forrest, & Zhao, 2011; Gong, Li, Zhou, & Yao, 2009; Jin, Ni, Chen, & Li, 2016; Liao & Xu, 2014a, 2014b; Liao, Xu, Zeng, & Merigó, 2015; Ureña, Chiclana, Fujita, & Herrera-Viedma, 2015; Wan, Wang, & Dong, 2016; Wang, 2013, 2015; Wu & Chiclana, 2014; Xu, 2007a, 2007b, 2012; Xu, Cai, & Szmidt, 2011; Xu & Liao, 2015; Xu, Wan, Wang, Dong, & Ze, 2016; Zeng, Su, & Sun, 2013). Just as other types of preference relations, consistency analysis is a necessary step to guarantee the reasonable ranking orders. At present, there are mainly two types of consistency concepts for IFPRs: additive consistency and multiplicative consistency (Gong et al., 2011; Liao & Xu, 2014a; Wang, 2013; Wu & Chiclana, 2014; Xu, 2007a, 2007b; Xu et al., 2011). According to the principles of these definitions, one can check that all of them are derived from the consistency

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concepts for reciprocal preference relations and fuzzy interval preference relations (FIPRs). Nevertheless, none of them is sufficient to address IFPRs. Based on these consistency concepts, many programming model-based methods to decision making with IFPRs have been developed, such as the linear goal programming methods (Gong et al., 2009; Gong et al., 2011; Jin et al., 2016; Wang, 2013; Xu, 2007b; Xu et al., 2016), the nonlinear programming methods (Behret, 2014; Liao et al., 2015), and the least squares programming methods (Gong et al., 2011; Wang, 2015). Furthermore, Zeng et al. (2013) applied the defined similarity measure to introduce a group decision-making method with IFPRs, and Wan et al. (2016) used the defined Hamming distance measure to give a group decision-making method with IFPRs. However, neither of them considers the consistency of individual IFPRs, this indicates that the unreasonable ranking order might be obtained. Xu (2012) presented an error-analysis-based method to deriving the interval priority weight vector from IFPRs. Considering the incomplete case, Xu et al. (2011) and Ureña et al. (2015) applied the multiplicative consistency concept in Xu et al. (2011) to give an interactive algorithm to determine missing values, respectively. However, as Liao and Xu (2014a) noted that this concept is too strict to define the consistency of IFPRs, and the contradictory conclusions might be derived with respect to the different compared orders of objects. Furthermore, Xu and Liao (2015) reviewed the main theoretical results about IFPRs before 2015. Besides the theoretical researches, the application of IFPRs has also received considerable attentions from researchers, such as the radio frequency identification (RFID) technology selection (Wan et al., 2016), the assessment of building clothing system (Behret, 2014), the selection of flexible manufacturing system (Liao & Xu, 2014a), the supply chain management (Xu, 2012), the selection of the locations for shopping center (Xu et al., 2011), the evaluation of professional title (Xu et al., 2016) and so on.

After reviewing previous researches about IFPRs, we find that all previous consistency concepts for IFPRs have limitations in some aspects. This leads to two serious results: one is that methods based on these concepts have theoretical drawbacks, and the other is that the ranking order obtained from these methods cannot guarantee rationality. Considering these issues, this paper continues to study IFPRs, and presents a multiplicative consistency and consensus based method to group decision making with IFPRs. To do this, we first review six multiplicative consistency concepts for IFPRs, and analyze their limitations. Then, we present a new multiplicative consistency definition by using preferred IFPRs (PIFPRs), which can address issues in previous ones. Using the new concept, 0-1 mixed programming models to judge the consistency and to determine missing values are constructed, respectively. After that, an algorithm to deriving the intuitionistic fuzzy priority weight vector is presented. Finally, a distance measure based consensus index is defined, and an improving consensus method is provided. The rest is organized as follows:

Section 2 first reviews several concepts about preference relations, such as reciprocal preference relations, IFPRs and FIPRs. Then, it recalls six multiplicative consistency concepts for IFPRs and analyzes their limitations in some aspects. Section 3 introduces a new multiplicative consistency concept for IFPRs using PIFPRs. Subsequently, 0-1 mixed programming models to judge the consistency of IFPRs are constructed. Section 4 focuses on the incomplete case, and establishes several consistency-based 0-1 mixed programming models to determine missing values. Then, an algorithm for deriving the intuitionistic fuzzy priority weight vector from IFPRs is developed. Section 5 gives a distance measure between any two individual IFPRs, by which a consensus index is obtained, and the weights of the DMs are determined. Then, an algorithm for group decision making with IFPRs is proposed. Section 6 contains two subsections. One subsection offers two practical examples to show the efficiency and feasibility of the new results, and the other analyzes the principles of several previous methods.

2. Preliminaries

This section contains two parts: the first part mainly introduces several basic notations and concepts to help us understand the following contents, and the other reviews several multiplicative consistency concepts for IFPRs and analyzes their limitations in some aspects.

2.1. Several basic concepts

Throughout the paper, let $X = \{x_1, x_2, \ldots, x_n\}$ denote the set of compared objects. To express the DMs' preferred degrees on objects in X, Tanino (1984) introduced the concept of reciprocal preference relations: A reciprocal preference relation R on X is defined by $R = (r_{ij})_{n\times n}$, where $r_{ij} \in [0,1]$ is the preferred degree or intensity of the object x_i over x_j, $r_{ij} = 0.5$ indicates indifference between x_i and x_j ($x_i \sim x_j$), and $r_{ij} > 0.5$ means that x_i is preferred to x_j ($x_i \succ x_j$). In general, $r_{ij} + r_{ji} = 1$ for all $i, j = 1, 2, \ldots, n$. To guarantee the reasonable ranking order, Tanino (1984) introduced the following multiplicative consistency concept for reciprocal preference relations.

Definition 1 Tanino, 1984. The reciprocal preference relation $R = (r_{ij})_{n\times n}$ is multiplicatively consistent if the following condition holds:

$$r_{ij}r_{jk}r_{ki} = r_{ji}r_{ik}r_{kj} \quad (1)$$

for all $i, k, j = 1, 2, \ldots, n$.

Let $w = (w_1, w_2, \ldots, w_n)$ be a weighting vector satisfying $\sum_{i=1}^{n} w_i = 1$ and $w_i \in [0,1]$ for all $i = 1, 2, \ldots, n$. Xu (2007b) considered that the elements of a multiplicative consistent reciprocal preference relation R can be expressed as follows:

$$r_{ij} = \frac{w_i}{w_i + w_j} \quad (2)$$

for all $i, j = 1, 2, \ldots, n$.

From the concept of reciprocal preference relations, one can find that it only permits the DMs to express their preferred information. When the DMs want to give their non-preferred judgments, reciprocal preference relations are helpless. To address this issue, Atanassov's intuitionistic fuzzy sets (IFSs) are good choices.

Definition 2 Atanassov, 1986. An IFS A on X is expressed as $A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle | x \in X\}$, where $\mu_A(x) \in [0,1]$ and $\upsilon_A(x) \in [0,1]$ are respective of the preferred and non-preferred degrees of the element $x \in X$ with the condition $\mu_A(x) + \upsilon_A(x) \leqslant 1$, and the hesitancy degree is denoted by $\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x)$.

Later, Xu (2007c) introduced the concept of intuitionistic fuzzy values (IFVs): An IFV $\tilde{\alpha}$ is expressed by $\tilde{\alpha} = (\mu, \upsilon)$, where $\mu \in [0,1]$ and $\upsilon \in [0,1]$ denote the preferred and non-preferred degrees with the condition $\mu + \upsilon \leqslant 1$, respectively. Xu (2007a) noted the advantages of IFSs to denote the DMs' information and introduced the concept of IFPRs:

Definition 3 Xu, 2007a. An IFPR on X is defined by a matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times n}$ such that $\tilde{r}_{ij} = (\mu_{ij}, \upsilon_{ij})$ is an IFV with $\mu_{ij} = \upsilon_{ji}$ and $\mu_{ii} = \upsilon_{ii} = 0.5$ for all i, $j = 1, 2, \ldots, n$, where μ_{ij} denotes the preferred degree of the object x_i over x_j, υ_{ij} indicates the preferred degree of the object x_j over x_i, and $\pi_{ij} = 1 - \mu_{ij} - \upsilon_{ij}$ is the indeterminacy degree between the objects x_i and x_j.