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Permutation flow shop scheduling problem to minimize nonlinear objective function with release dates

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ABSTRACT

In a dynamic flow shop scheduling model, each released job has to be executed on a set of machines in series following the identical route. The criterion of optimality, total k-power completion time ($k \geq 2$) is addressed for this problem. Given its NP-hardness, a branch and bound (B&B) algorithm is designed to obtain the optimal solutions for small-scale problems. On the basis of an initial population generated by the upper bounds of the B&B algorithm, a discrete differential evolution algorithm is introduced to solve medium-scale problems. As the by-product of the B&B algorithm, a sequence-independent lower bound with performance guarantee is presented as a substitute of the optimal solution for large-scale problems. Moreover, the worst-case ratio of the shortest-processing-time-based rule under a consistency condition is provided for the problem. The numerical experiments demonstrate the effectiveness of the proposed algorithms and the new lower bound.

1. Introduction

The flow shop scheduling is a familiar model in manufacturing industry. For example, coronary stent production usually consists of four phases: First, a laser cutter engraves the raw material, titanium alloy tube, under inert gas protection. Second, the semi-finished stent is pickled in acidic solution to remove the oxide layer on the surface. Third, a sanding machine grinds the stent surface for deburring. Finally, the stent is polished electrochemically to enhance the surface smoothness further. The objectives include the minimization of machine loads and work-in- process inventory. Academically, this process can be modelled as a dynamic flow shop scheduling problem to optimize makespan (*i.e.* maximum completion time, C_{max}) and total completion time (TCT, ΣC_j). A classical method of handling the bi-objective scheduling problem is to linearize the two criteria with a constant factor (*i.e.* $\alpha C_{\max} + (1-\alpha)\Sigma C_j,\ \ 0 \leq \alpha \leq 1$). However, determination of an appropriate factor is quite difficult to express the original bi-criterion. Cheng and Liu (2004) reported that the non-linear objective, total k-power completion time (TKCT, ΣC_j^k), builds a coordinated relationship between the criteria: makespan ($k \to \infty$) and total completion time ($k = 1$). Industrially, the criterion of makespan decreases machine loads and economizes energy consumption, whereas TCT reduces work-in-process and saves on inventory cost. This paper selects the TKCT objective for the dynamic flow shop scheduling model to serve as a trade-off between energy consumption and inventory cost.

Bai (2015) presented the NP-hard result for the flow shop total quadratic completion time (TQCT) problem with release dates, which indicates that the dynamic flow shop TKCT ($k \geq 2$) problem is unsolvable in polynomial time, and the optimal solution can only be achieved by using enumeration-based methods, such as branch and bound (B&B) algorithm. To the best of the authors' knowledge, researches that use B&B algorithm to solve the static flow shop model mainly focus on the linear version (*i.e.*, k = 1) of the TKCT objective. Such researches include Ignall and Schrage (1965), Bansal (1977), Ahmadi and Bagchi (1990) and Chung, Flynn, and Kirca (2002), and no study has addressed on the non-linear version (*i.e.*, $k \geq 2$) with release dates. Compared with the static model that is convenient for the study, the dynamic model more accurately simulates the industrial production environment. This study designs an efficient B&B algorithm to optimize the flow shop scheduling problem at a small scale. In this algorithm, a branching rule that is suitable for all dynamic flow shop models and a lower bound based on the shortest remaining processing time first (SRPT) rule for single-machine problem significantly reduce the size of the search space. However, because of the enumerative nature, finding the optimal solution with the B&B algorithm may take a

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prohibitively long time as problem scale increases. Given that meta-heuristic algorithm is more efficient to achieve high-quality solution within a specified time for medium-scale problems, a discrete differential evolution (DDE) algorithm is introduced to obtain near-optimal solution with an initial population generated by the upper bounds of the B&B algorithm. In addition, a sequence-independent lower bound with performance guarantee and the worst-case ratio of the shortest processing time available (SPTA)-average heuristic under the consistency condition are provided for the problem. A series of numerical experiments demonstrate the performance of the proposed algorithms and the new lower bound.

The remainder of this paper is organized as follows: Section 2 briefly surveys the nonlinear objective and DDE algorithm for the flow shop scheduling problems. Section 3 presents the mathematical formulation for the problem. Sections 4 and 5 propose the B&B and DDE algorithms for the problem, respectively. Section 6 proves the worst-case ratio for the SPTA-average heuristic and the asymptotic optimality for the new lower bound. Section 7 executes numerical experiments for the algorithm and lower bound. Section 8 concludes the paper. The experimental results for determining the parameters of the DDE algorithm are provided in Appendix.

2. Literature review

To describe the scheduling problems, formally, the standard three-field notation (Graham, Lawler, Lenstra, & Rinnooy Kan, 1979) is employed in the following discussion.

2.1. Nonlinear objective in flow shop scheduling

Unlike the linear objective, only a few studies have focused on the flow shop scheduling problems with nonlinear criterion. Koulamas and Kyparisis (2005) reported the NP-hardness of the $Fm||C_j^2$ problem, and determined the asymptotic optimality and worst-case ratio of the shortest processing time first (SPT) heuristic. Xu, Sun, and Gong (2008) executed worst-case analysis on the SPT heuristic for the flow shop TQCT problem with linear and power learning-effects. Wang and Wang (2012) extended the worst-case performance of the SPT heuristic to the flow shop TQCT problem with exponential learning-effect. Bai (2015) indicated the asymptotic optimality of the SPTA-based heuristic for the $Fm|r_j|C_j^2$ problem. Bai and Zhang (2014) generalized the asymptotic optimality of the SPT- and SPTA-based heuristics to the $Fm||C_j^k$ and $Fm|r_j|C_j^k$ problems, respectively. For the $Fm||w_jC_j^2$ problem, Ren et al. (2017) performed asymptotic and worst-case analysis on the weighted SPT-based heuristic, and presented a DDE algorithm to obtain near-optimal solutions. The known studies mainly analyze the performance of the SPT- or SPTA-based heuristics in theory. However, these heuristics do not work well for small-scale problems in spite of their convergence property. This paper designs a B&B algorithm to achieve optimal solution in which a release-date-sensitive branching rule can efficiently cut invalid nodes.

2.2. DDE algorithm in flow shop scheduling

Differential evolution (DE) is a novel population-based search algorithm for global optimization over continuous domains (Storn & Price, 1997). This simple and efficient algorithm has successfully handled many numerical optimization problems, outperforming other popular evolutionary meta-heuristics. However, the canonical DE algorithm cannot directly solve discrete optimization problems because of its continuous nature. With a discretization procedure (*i.e.* converting the floating-point solution vectors into discrete-valued vectors), therefore, a discrete DE (DDE) algorithm is proposed to deal with scheduling problems. The relative research works that use the DE-based algorithms for flow shop scheduling problems are summarized in Table 1. A common characteristic of these algorithms is the initial population that is randomly generated without performance guarantee. This paper generates the initial population of the DDE algorithm with the upper bounds of the B&B algorithm, which ensures the high-quality initial solutions and accelerates solving process.

Table 1
Summary of the DE-based algorithms for the flow shop scheduling problems.

Problem model	Algorithm	Reference
Permutation flow shop	DDE	Onwubolu and Davendra (2006)
	DDE with local search	Pan, Tasgetiren, and Liang (2008)
	Hybrid DDE	Li and Yin (2013)
		Liu, Yin, and Gu (2014)
	Self-guided DE	Shao and Pi (2016)
	Algebraic DE	Santucci, Baioletti, and Milani (2016)
Permutation flow shop with no-idle	DDE with local search	Deng and Gu (2012)
	Hybrid DDE	Tasgetiren, Pan, Suganthan, and Buyukdagli (2013)
Permutation flow shop with blocking	Hybrid DDE	Qian, Wang, Huang, Wang, and Wang (2009)
	DDE	Wang, Pan, Suganthanc, Wang, and Wang (2010)
	Hybrid DDE	Han, Gong, and Sun (2015)
Permutation flow shop with no-wait	DDE with local search	Pan, Wang, and Qian (2009)
Assembly flow shop with setup times	Self-adaptive DE	Al-Anzi and Allahverdi (2007)
Batch flow shop with blocking and release dates	Hybrid DDE	Chen, Zhou, Li, and Xu (2014)

3. Mix integer programming model

A flow shop scheduling model can be formally expressed as follows: A set of n jobs is processed on m machines in series. Job j, $j = 1, 2, \ldots, n$, has a non-negative processing time $p_{i,j}$ on machine i, $i = 1, 2, \ldots, m$, and a release date r_j that is the earliest time when the job is available. The machine processes the jobs on a first-come, first-served manner, and the jobs go through each machine in identical order (permutation schedule). The intermediate storage between successive machines is unlimited. Preemption is forbidden, *i.e.*, any commenced operation has to be completed without interruptions. The objective is to gain a schedule to optimize the criterion of TKCT ($k \geq 2$). The following notations are listed for the expression of the scheduling model:

N	$= \{1, 2, \ldots, n\}$, the set of jobs to be scheduled.
M	$= \{1, 2, \ldots, m\}$, the set of machines in the shop.
$S_{i,j}$	= the starting time of job $j \in N$ on machine $i \in M$.
$C_{i,j}$	= the completion time of job $j \in N$ on machine $i \in M$.
C_j	$= C_{m,j}$, the completion time of job $j \in N$ on the last machine.
$X_{j,j'}$	$= \begin{cases} 1, & \text{job } j \text{ precedes job } j' \text{ immediately}, j \neq j'; \\ 0, & \text{otherwise}. \end{cases}$
Y	= a very large positive number

With the concept of one-job-one-position, the $Fm|r_j|\Sigma C_j^k$ problem can be formally expressed as the following mathematical programming model: