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An efficient Wave Based Method for solving Helmholtz problems in three-dimensional bounded domains

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ABSTRACT

This paper discusses the use of the Wave Based Method for the analysis of time-harmonic three-dimensional (3D) interior acoustic problems. Conventional element-based prediction methods, such as the Finite Element Method, are most commonly used for these types of problems, but they are restricted to low-frequency applications. The Wave Based Method is an alternative deterministic technique which is based on the indirect Trefftz approach. Up to now, this method's very high computational efficiency has been illustrated mainly for two-dimensional (2D) problem settings, allowing the analysis of problems at higher frequencies. The numerical validation examples presented in this work shows that the enhanced computational efficiency of the Wave Based Method in comparison with conventional element-based methods is kept when the method is extended to 3D case with and without the presence of material damping.

1. Introduction

In the context of optimising the design of mechanical systems, the acoustical and vibrational comfort of a product has taken up a predominant place among more traditional design criteria such as strength, durability, maintainability, etc. With the development of modern computer systems, the exponential increase in computational power and the availability of advanced and robust simulation techniques, the use of detailed numerical models for functional performance evaluation has become an indispensable part of many contemporary design processes. The use of these so-called *virtual prototypes* significantly reduces the time and the cost needed to assess a single design possibility and allows designers to predict the behaviour of their products without the need to construct physical prototypes. In this way, the number of design alternatives which can be fully explored increases and a better assessment of the performance impact of several design options can be made prior to taking important and irreversible design decisions.

In the scope of analysing the interior acoustic behaviour of products and processes, element-based approaches such as the Finite Element Method (FEM) and the Boundary Element Method (BEM) are by far the most commonly applied deterministic numerical prediction techniques for the analysis of time-harmonic acoustic problems.

In the FEM [1], the entire problem domain is divided into a (often very large) number of discrete elements. Within these elements, the acoustic pressure field is approximated using a superposition of simple, usually lower-order polynomial shape functions. As the excitation frequency increases, however, an increasingly refined discretisation is needed to suppress the associated interpolation and pollution errors [2] due to the approximative nature of the applied shape functions. This requirement results in very large numerical models, the solution of which involves a prohibitively large amount of computational resources. Hence, the FEM is limited to low-frequency applications [3].

As compared to the FEM, the BEM [4] reformulates the governing Helmholtz partial differential equation as a mathematically equivalent boundary integral formulation of the problem, such that only the boundary of the considered domain has to be discretised. Within each boundary element, the distributions of the boundary variables are approximated using an expansion of locally defined simple, polynomial shape functions. Enforcement of the boundary conditions results in a small numerical model, as compared to FE models, which can be solved for the nodal values on the discretised boundary. Once these nodal values are known, the field variables inside the domain may be reconstructed by applying the boundary integration formulations in a postprocessing step. As compared to the fast assembly of frequency-independent, real valued, sparse FE matrices, the construction of the frequency-dependent, complex, densely populated BE matrices is very time consuming. In this way, the smaller model size does not necessarily result in an enhanced computational

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efficiency, so that the practical use of the BEM is also restricted to low-frequency applications [5].

Whereas the FEM, due to the domain partitioning into small elements, is more suited for the analysis of problems in bounded domains – although there are several techniques which allow the analysis of problems in unbounded media [6,7] – the BEM is more applicable to the latter class of problems, but can hardly compete with the FEM for solving interior acoustic problems. However, some recent developments which increase the computational efficiency of the BEM, such as the fast multi-pole BEM (FEMBEM) [8,9], the wave boundary elements [10,11] and the wave number independent BEM [12–14], may prove to make the BEM more competitive for the analysis of bounded problems.

In recent years, a vast amount of research efforts has been spent on the development of possible extensions of the FEM in order to minimise or even eliminate the pollution error and, as a result, increase the practical application range of the method to higher frequencies. Process optimisation techniques, such as adaptive FE refinement [2,15,16], reduced numerical integration [17–20] and efficient numerical solvers [21,22], do not modify the basic FE formulations, but focus on the optimisation of the numerical process in order to obtain an enhanced computational efficiency.

Another way to reduce the computational efforts involved with solving large FE problems is by applying domain decomposition techniques, such as component mode synthesis [23], automated multi-level substructuring [24,25] and finite element tearing and interconnecting [26,27]. Due to the application of a *divide and conquer* strategy, these domain decomposition techniques are perfectly suited for implementation in a parallel computing environment.

Stabilised FE methods, such as the Galerkin least squares FEM [28], the Galerkin gradient least squares FEM [29] and the quasi-stabilised FEM [30], reduce the pollution error of the FEM by modifying the weak form of the integral problem formulation.

Instead of modifying the integral problem formulation to reduce the pollution error, the family of generalised methods tries to reduce the pollution error by introducing *a priori* knowledge about the solution in the numerical FE scheme through the enrichment of the conventional polynomial shape function basis. Two such methods are the partition-of-unity FEM, in the literature also referred to as the generalised FEM [31,32], and the element-free Galerkin method [33,34]. The ultra weak variational formulation [35,36] also incorporates a priori known information of the solution in the numerical scheme but embeds it into a novel variational formulation.

Another class of improved FEMs are the so-called multi-scale methods. These methods consider the solution to be a superposition of a large scale and a fine scale component. The large scale component is usually approximated with polynomials, while for the small scale component wave-like functions are applied. The wave-like functions incorporate a priori known information about the solution. Methods which are classified as multi-scale methods are the discontinuous enrichment method [37] and the related discontinuous Galerkin method [38].

Many of these improvements to the FEM have recently also found their way into the BEM research community. In this way, the BEM and its derivations like the FMBEM can also profit from the increased computational efficiency provide by e.g. dedicated preconditioned iterative solvers [39], multi-domain interface treatment [40], domain decomposition methods [41,42] and the use of hierarchical matrices [43,44].

Apart from the FEM, BEM and all the methods derived from their basic concepts, there is another family of methods, the so-called Trefftz methods [45], which distinguish themselves from the FEMs by their choice of shape and weighting functions [46]. Instead of applying approximation functions, exact solutions of the governing differential equations are used for the expansion of the field variables. Examples of such Trefftz-based methods which have been recently applied for the study of time-harmonic acoustic and structural dynamic problems are the Wave Based Method (WBM) [47], the Variational Theory of Complex Rays (VTCR) [48–50] and the Method of Fundamental Solutions [51,52]. Since the functions which are applied in the WBM to expand the dynamic pressure field are exact solutions of the governing Helmholtz equation, no residual error is involved with respect to the governing partial differential equation inside the problem domain.[1] However, the functions may violate the boundary conditions. By minimising the residuals of the boundary conditions in a Galerkin weighted residual formulation, a small system of algebraic equations is obtained, which can be solved for the contribution factors of the expansion functions. Due to the small model size and the enhanced convergence characteristics of the WBM, it has a superior numerical performance as compared to the FEM. As a result, problems at higher frequencies can be addressed. The WBM has been applied successfully for the analysis of two-dimensional (2D) interior and exterior (vibro-) acoustic problems [46,53,54], for the structural dynamic analysis of flat plates [55,56] and for the analysis of poroelastic materials [57]. Recent research has focussed on the treatment of semi-infinite problems [58] and on the extension of the geometrical flexibility of the method through the use of spline-based boundary definitions [59], the combination of the WBM and FEM in a hybrid FE–WBM [60–62] and the development of a novel multi-level modelling approach for multiple scattering and inclusion problems [63,64]. The principles underlying the WB modelling paradigm have also been applied to efficiently model three-dimensional (3D) interior vibro-acoustic systems [65] and exterior acoustic scattering and radiation problems [66]. The main focus of the research in the former paper is on the theoretical derivation of multi-physical vibro-acoustic coupling formulation in fully Trefftz-based numerical models and the analysis of the stability thereof. The aim of the second work proposes a suitable and effective set of 3D acoustic wave functions for the description of the propagation of sound fields in unbounded acoustic media exterior to a spherical truncation geometry. In this paper, these developments are complemented by an in-depth analysis of the properties of a WB modelling approach for 3D interior acoustic problem domains. Through a comparison of this method with linear and quadratic FE approaches, the method's computational efficiency and its potential for mid-frequency numerical analysis of both damped and undamped acoustic cavities is revealed. Moreover, an empirically derived general guideline for determining the required number of WB approximation functions as a function of the problem geometry and the analysis frequency is presented.

Section 2 introduces a mathematical description of a time-harmonic acoustic problem. Section 3 introduces the WBM methodology and lists the formulations for interior 3D acoustic problems. Section 4 compares the properties of the WBM with those of classical element-based modelling techniques and highlights the main advantages and limitations of the methodology. Section 5 illustrates the beneficial properties of the proposed method for the study of the acoustic behaviour of a 3D acoustic cavity.

2. Definition of a Helmholtz problem in a bounded domain

Consider a steady-state bounded acoustic problem, as shown in Fig. 1. A closed boundary surrounds a bounded fluid domain V,

[1] It should be noted here that stating that there is no residual error with respect to the governing partial differential equation does not mean that there is no error inside the problem domain. However, the error inside the domain is purely related to the error on the boundary conditions and not due to a violation of the governing equation.