Computers & Industrial Engineering 111 (2017) 364–380

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A versatile adaptive aggregation framework for spatially large discrete location-allocation problems

Matej Cebecauer [a,*], L'uboš Buzna [b,c]

[a] Department of Transport Science, KTH Royal Institute of Technology, Teknikringen 10, SE-100 44 Stockholm, Sweden
[b] Department of Mathematical Methods and Operations Research, University of Žilina, Univerzitná 8215/1, SK-010 26 Žilina, Slovakia
[c] ERA Chair for Intelligent Transport Systems, University of Žilina, Univerzitná 8215/1, SK-010 26 Žilina, Slovakia

ARTICLE INFO

Article history:
Received 20 December 2016
Received in revised form 11 June 2017
Accepted 16 July 2017
Available online 18 July 2017

Keywords:
Data aggregation
Location analysis
Adaptive aggregation
Framework
Heuristics

ABSTRACT

We propose a versatile concept of the adaptive aggregation framework for the facility location problems that keeps the problem size in reasonable limits. Most location-allocation problems are known to be NP-hard. Thus, if a problem reaches the critical size, the computation exceeds reasonable time limits, or all computer memory is consumed. Aggregation is a tool that allows for transforming problems into smaller sizes. Usually, it is used only in the data preparation phase, and it leads to the loss of optimality due to aggregation errors. This is particularly remarkable when solving problems with a large number of demand points. The proposed framework embeds the aggregation into the solving process and it iteratively adjusts the aggregation level to the high quality solutions. To explore its versatility, we apply it to the p-median and to the lexicographic minimax problems that lead to structurally different patterns of located facilities. To evaluate the optimality errors, we use benchmarks which can be computed exactly, and to explore the limits of our approach, we study benchmarks reaching 670,000 demand points. Numerical experiments reveal that the adaptive aggregation framework performs well across a large range of problem sizes and is able to provide solutions of higher quality than the state-of-the-art exact methods when applied to the aggregated problem.

1. Introduction

1.1. Motivation

High quality design of public service systems or private supply chains is of significant importance due to tight public budgets and today's highly competitive globalised environment, as it has the potential to facilitate collaboration mechanisms to build innovative partnerships leading to new types of businesses and services. To ensure efficient public spendings and to reach low transportation costs and high performance of service systems, analytical tools able to provide an effective decision support are required. Three main pillars of a decisions support tool are the model that captures important aspects of a real-world system, the efficient and flexible solving method and high fidelity input data. The field data describing a system's operation and the relevant characteristics of external environment are today more available than ever, due to the recent progress in ICT technologies enabling new concepts of sensing, data transmission, data storage and data processing, and due to open data initiatives enabling data sharing. Most approaches to the design of public service systems or supply chains focus on the development of new models or solving methods. In contrast, this paper is focused on the input data, and it examines the potential of how aligning aggregation of the input data with high quality solutions could help increase the quality of solutions provided by a decision support tool when data aggregation is unavoidable due to the high demand of solving methods on computational resources.

1.2. Review of literature on location-allocation problems

The need to design a public service system or a private supply chain often leads to a location-allocation problem. Researchers have been formulating and studding location-allocation problems for decades, and they have recognized a variety of forms depending on the particular application, objectives and constraints (Daskin, 1995; Drezner, 1995; Eiselt & Marianov, 2011, 2015).

Two basic problem classes are formed by continuous location problems where facilities are located in a plain or in some other type of a continuous set, and discrete location problems, where locations of facilities are selected from a finite set of options.

* Corresponding author.
E-mail addresses: matejc@kth.se (M. Cebecauer), Lubos.Buzna@fri.uniza.sk (L'. Buzna).

http://dx.doi.org/10.1016/j.cie.2017.07.022
0360-8352/

According to the type of the objective, it is common to differentiate between minisum problems (minimising the sum of customers' utilities), mimimax (minimising the maximum customer's utility), and covering problems (ensuring that either each or the maximum number of customers' utilities reach the pre-defined quality) (Eiselt & Marianov, 2011). Nowadays, most problems are optimized with respect to a multicriterial objective function. The examples include bi-objective and k-objective location problems that incorporate operational goals such as total setup costs, fixed costs, average time/distance travelled, number of located facilities, fuel consumption, or also more recently, environmental and social goals based on land use, congestion, noise, pollution or tourism (Doerner, Focke, & Gutjahr, 2007; George & ReVelle, 2003; Hamacher, Labbé, Nickel, & Skriver, 2002; Nickel, Puerto, Rodríguez-Chía, & Weissler, 2005). For a comprehensive overview of multicriterial location problems, see Farahani, SteadieSeifi, and Asgari (2010). Similarly, various approaches are considered as the primary attractiveness determinant when allocating a customer to the located facilities. The basic approach is to associate customers with a single, e.g. the closest (Hakimi, 1965), or with multiple facility locations (Achabal, 1982). The approach by Huff (1962), allocates a share of customer's demand to all located facilities using the gravitational force formula. Another well-known model is the multinomial logit model (Gupta, Chintagunta, Kaul, & Wittink, 1996). More complex approaches consider several determinants simultaneously, e.g. the travel time and the waiting time (Marianov, Ríos, & Icaza, 2008). If customers do not have knowledge of the functionality of facilities, they may chose to follow the strategy of visiting a number of pre-assigned facilities until they acquire the service or give up trying after a given number of unsuccessful trials (Yun et al., 2015).

Over the last few decades, the location research has addressed also many modelling challenges directly induced by practical applications. Among them we mention efforts aiming at capturing uncertainties in the operation of a system and the development of complex, but tractable formulations of problems that combine strategic location decisions with tactical and operational decisions associated with the organization of services and flows. In the context of the supply chain network, the uncertainty concerns both the demand and the supply (Snyder & Daskin, 2005). The authors in An, Zeng, Zhang, and Zhao (2014) consider a robust optimization model, where k facilities may fail. The problem is optimized with respect to the multicriterial objective function aiming at finding a trade-off between the operational costs of the least costly and the most costly disruptive scenarios. The research paper Fei and Mahmassani (2011) is an example of the demand uncertainty minimisation for optimal sensor locations where a multi-objective problem maximising the link information gains in the conjunction with the demand coverage applying hybrid greedy randomized adaptive search procedure to identify the Pareto frontier is presented. An early attempt to address the combined location-routing problem is the work by Perl and Daskin (1985). Ouyang, Wang, and Yang (2015) presented a modelling approach for the median type of facility location under elastic customer demand and traffic equilibrium in a continuous space. The study Ponboon, Qureshi, and Taniguchi (2016) formulates a mathematical model, and it proposes a solving method based on the branch and price (column generation) algorithm for the location routing problem with time windows. Romero, Nozick, and Xu (2016) developed a model to analyse the facility location and routing in the context of hazardous materials transportation. Detailed review on location-routing problems is provided in Prodhon and Prins (2014).

The proposed adaptive aggregation framework is broadly applicable. To evaluate it, we selected two problems. An archetypal example for a location-allocation problem is the *p-median problem* (Hakimi, 1965). The number of algorithms applicable to this problem available in the literature is large. One of the most successful exact algorithms is ZEBRA (García, Labbé, & Marín, 2011). ZEBRA is based on the radial formulation of the p-median problem and for a narrow range of located facilities allows for solving problems up to 80 000 demand points. Thus, problems of larger size are solved by classical heuristics or metaheuristics. Examples of classical heuristics include: greedy heuristics that grow the number of located facilities one by one (Whitaker, 1983), dual accent heuristics based on the dual of the relaxed integer programming formulation of the problem (Erlenkotter, 1978), interchange heuristics where facilities are moved iteratively to vacant sites if it reduces the value of the objective function (Teitz & Bart, 1968) and the alternate heuristic (ALA) (Cooper, 1964, 1972; Maranzana, 1964). In the first step of ALA heuristic customers are divided into p subsets, and single optimal location is determined for each group. In the second step, allocations of customer are re-optimized. Location and allocation steps are alternated until no further improving changes are possible. In the literature there are several papers proving the convergence of ALA heuristic (Drezner, 1992; Lawrence & Ostresh, 1978). For comprehensive overview of heuristic and metaheuristic algorithms to the p-median problem please refer to Mladenović, Brimberg, Hansen, and Moreno-Pérez (2007). The second selected problem is the *lexicographic minimax facility location problem* (Ogryczak, 1997). This problem is motivated by the need arising in some applications to consider the equitable access of customers to located facilities. The goal is to find the location of facilities that corresponds to the lexicographically smallest non-increasingly ordered vector of disutilities that are associated with allocations of customers to facilities. The vector can be rearranged, where the k-th term in the vector is the number of occurrences of the k-th worst possible unique outcome (disutility). The optimal solution is then found by minimising the value of the first vector element followed by the minimisation of the second element without worsening the first term and so on (Ogryczak, 1997). As an alternative, the ordered outcomes approach and the ordered values approach were proposed in Ogryczak and Śliwiński (2006). The latter approach is more efficient, however, the size of solvable instances is very small. A convenient technique for interactive analysis, where facilities are located with respect to the objective function taking into account lexicographic minimax combined with the minisum term, was proposed in Ogryczak (1999). The approach is based on the reference distribution method which can be steered by manipulating few parameters only and allows to take into account aspiration values of assigned distances defined by the user. The above mentioned approaches to the lexicographic minimax optimization result in a specific form of the mathematical model that is supposed to be solved by a general purpose solver. This limits the size of solvable problem to less than 1000 demand points. Approximative algorithm ALEX that provides high quality solutions and enables to extend the size of solvable problems to several thousands was proposed in Buzna, Koháni, and Janáček (2014).

1.3. Review of literature on aggregation errors

Aggregation methods have been a subject of intense research in the fields of transport economics, operations research and geographic information systems. They lead to problems of smaller size where the demand points (DPs) are replaced by the aggregated demand points (ADPs). The simplest aggregation strategies are dividing geographical space using regular grid (Hillsman & Rhoda, 1978) or selecting randomly a sample of demand points that represent them all (Goodchild, 1979). As an alternative, Erkut and Bozkaya (1999) describes the iterative aggregation of pairs of DPs that are close to each other based on a defined measurement. In contrast to the previous methods, the row-column