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Linguistic multi-attribute decision making with multiple priorities

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ABSTRACT

Linguistic multi-attribute decision making (MADM) problems have received many attentions in theoretical and practical aspects. It is well known that center of the linguistic MADM is the aggregation of fuzzy linguistic information. As one widely used type of aggregation operators, the importance weighted aggregation assumes that all the attributes are at the same priority level. This paper deals with linguistic MADM with multiple priorities in terms of strict priority hierarchy or weak priority hierarchy. To do so, based on the 2-tuple linguistic model and the revised product t-norm, a prioritized linguistic idempotent scoring (PLIS) operator is first proposed for linguistic MADM with a strict priority hierarchy to guarantee the idempotent property of prioritized aggregation. Moreover, as an extension of the PLIS operator, a PLIS ordered weighted averaging (PLIS-OWA) operator is proposed to solve linguistic MADM with a weak priority hierarchy. In addition, in practice decision-makers (DMs) are often unsure of their evaluations due to time pressure, lack of experience and data, and therefore may provide uncertain linguistic assessments. The PLIS and PLIS-OWA operators are also extended to the cases of prioritized MADM with interval-valued linguistic 2-tuples. Finally, three illustrative examples and comparative analysis are provided to show the effectiveness and efficiency of the proposed aggregation operators. The proposed operators outperform existing studies in terms of the idempotent property, no information loss, and heterogeneous information.

1. Introduction

Multi-attribute decision making (MADM) is concerned with ranking a finite number of decision alternatives, each of which is explicitly described in terms of different attributes (also called criteria or objectives) (Yan, Huynh, Ma, & Nakamori, 2013a). In practice, MADM often requires subjective assessments to solve complex and unstructured problems (Yan, Ma, & Huynh, 2016), which are often vaguely qualitative and cannot be estimated by exact numerical values. In this sense, the fuzzy linguistic approach (Zadeh, 1975a, 1975b, 1975c) enhances feasibility, flexibility, and reliability of decision models when decision problems are too complex or ill-defined to be described properly by conventional quantitative expressions (Herrera & Herrera-Viedma, 2000). By scanning the literature, one can find extensive applications of fuzzy linguistic approaches to many different areas such as new product development (e.g., Huynh & Nakamori, 2011; Yan & Ma, 2015a), quality function deployment (e.g., Wang, Fung, Li, & Pu, 2016; Yan, Ma, & Li, 2013b; Yan & Ma, 2015b; Yan, Ma, & Huynh, 2014), supply chain management (e.g., Karsak & Dursun, 2015; Meng, Lou, Peng, & Prybutok, 2016), and energy planning (e.g., Doukas, 2013; Sánchez-Lozano, García-Cascales, & Lamata, 2015; Yan, 2016).

It is well known that the fuzzy linguistic approaches follow a common resolution scheme consisting of two phases (Herrera & Martínez, 2000; Rodríguez & Martínez, 2013): aggregation and exploitation, center of which is the aggregation of fuzzy linguistic information, which heavily depends on the semantic description of the linguistic term set (Yan et al., 2016). On one hand, three categories of models have been proposed in the literature (Rodríguez & Martínez, 2013): the approximate model based on the extension principle (e.g., Yan & Ma, 2015a), the symbolic model (Bordogna, Fedrizzi, & Pasi, 1997), and the 2-tuple linguistic model (Herrera & Martínez, 2000). Perhaps due to its no information loss, straightforwardness and convenience in calculation, the 2-tuple linguistic model (Herrera & Martínez, 2000) has been widely studied in the literature (Herrera, Alonso, Chiclana, & Herrera-Viedma, 2009; Martínez & Herrera, 2012) and will also be used in this study. On the other hand, the issue of aggregation in fuzzy linguistic decision has been studied extensively in the literature (e.g., Ben-Arieh, 2005; Herrera, Herrera-Viedma, & Verdegay, 1996; Merigó,

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Palacios-Marqués, & Zeng, 2016; Wei, 2011; Yan et al., 2016). Among these studies, it is important to consider different importances of different attributes since some attributes are more important than others. In this case, decision-makers (DMs) may associate different importance weights with different attributes. These aggregation operators work well in situations where any difference is viewed as being in conflict because the operators reflect a form of compromise behavior among different attributes (Yan, Huynh, Nakamori, & Murai, 2011), i.e., the importance weights associated with attributes play a fundamental role in comparison among alternatives by overseeing tradeoffs among respective satisfactions of different attributes (Yager, 2004, 2008).

The importance weighted aggregation assumes that all the attributes are at the same priority level, which may be infeasible in real world decision problems. A typical example concerns the attributes of *safety* and *cost* in the cases of buying a car (Yan et al., 2011), selecting a bicycle for children (Yager, 2008) or air travel (Yager, 2004). In such cases, we usually do not allow a loss in *safety* to be compensated by a benefit in *cost*, i.e., tradeoffs between *safety* and *cost* are unacceptable. Simply speaking, the attribute *safety* has a higher priority than *cost*. Moreover, there may exist priority relationships among attributes in information retrieval (Yager, 2008). For example, a user intends to look for literature about decision making and prefers if they were written after 2003. In this case, the condition about decision making has a priority, because the user will not be interested if a paper (or book) is not about the topic. Over the past decade, we have witnessed many studies focusing on prioritized MADM with crisp numerical values. For example, Yager (2004) introduced an ordered weighted averaging (OWA) prioritized aggregation operator, where the weight of an attribute is determined by the OWA weighting vector together with the satisfactions of the attributes with a higher priority. For recent research about prioritized aggregation of crisp numerical values, please refer to Chen and Wang (2009), Yager (2008), Yan et al. (2011) and Yu, Xu, and Liu (2013).

Several researchers have studied prioritized aggregation in fuzzy linguistic MADM problems. On one hand, some researchers focus on prioritized aggregation based on the symbolic model. For example, Yager (1991) introduced non-monotonic operators of intersection and union which preserve priorities among arguments expressed as fuzzy subsets. Yager (1992) utilized the non-monotonic intersection operator to present the so-called second order attribute, such as "I want a good job, near my house if possible". Yager (1998) used two different methods, the weighted conjunction of fuzzy sets and fuzzy modeling, to develop the operators for fusion of fuzzy information. Chen, Xu, and Ma (2005) extended the non-monotonic intersection operator (Yager, 1991) to a prioritized information fusion method. Yager (2007) proposed a number of approaches, such as conjunction operators and disjunction operators, to aggregate ordinal information. On the other hand, there are also some studies based on the fuzzy extension principle. For example, Zhao, Lin, and Wei (2013) developed some prioritized aggregation operators for aggregating triangular fuzzy information. Yu and Xu (2013) introduced the concepts of a prioritized intuitionistic fuzzy aggregation operator and a prioritized intuitionistic fuzzy OWA operator. Li and He (2013) developed intuitionistic fuzzy prioritized "and" and "or" operators.

Despite the great advancements in linguistic MADM with multiple priorities, there are still three limitations in the literature. First, the prioritized aggregation operators based on the symbolic model make computations on the symbols of linguistic terms. The results yielded by such operators do not exactly match any of the initial linguistic terms and may cause loss of information (Herrera & Martínez, 2000; Rodríguez & Martínez, 2013). As pointed out by Yager (1998), such prioritized aggregation operators are "less sensitive" to slight changes in the satisfaction values of attributes. The approximate model based operators make operations on the fuzzy numbers, will create the burden of quantifying a qualitative concept (Herrera & Martínez, 2000) and complex mathematical computations (Yan et al., 2013b). As mentioned by Herrera and Martínez (2000) and Rodríguez and Martínez (2013), the results yielded by the approximate model do not exactly match any of the initial linguistic terms, so a process of linguistic approximation must be applied. This process causes loss of information and hence a lack of precision. Second, given an alternative with the same satisfaction value for all the attributes, intuitively the overall satisfaction value of the alternative will be the same as that of each attribute, no matter whether a prioritization of attributes exists or not. It is referred as the idempotent property. However, the scoring operators proposed by Yager (2008) do not guarantee such a property. Finally, in some cases, the DMs may have a set of possible linguistic terms about the attributes or alternatives (Yan et al., 2016). In this sense, the DMs could express their evaluations with interval-valued linguistic 2-tuples (Zhang, 2012, 2013), which is missed in the literature of prioritized aggregation.

Towards this end, this paper tries to propose alternative aggregation operators for prioritized MADM with single linguistic terms and interval-valued 2-tuples, so as to overcome the above limitations. The main contributions of this paper are twofold. First, we propose two novel prioritized aggregation operators for linguistic MADM with strict and weak priority hierarchies. Second, these two prioritized aggregation operators have been extended to the cases of prioritized MADM with interval-valued 2-tuples. The rest of this paper is as follows. Section 2 briefly reviews some basic knowledge used in this study. A prioritized aggregation operator is proposed for linguistic MADM with a strict priority hierarchy in Section 3, and then is extended to the case with a weak priority hierarchy in Section 4. Section 5 extends these two operators to prioritized MADM with interval-valued 2-tuples. Finally, some concluding remarks are provided in Section 6.

2. Preliminaries

2.1. Triangle norms

A triangle norm (t-norm) T is a mapping from $[0,1]^2$ to [0,1], which is increasing in both arguments. T-norm satisfies the properties of commutativity, monotonicity and associativity. It also fulfills the boundary condition: $\forall x \in [0,1], T(x,1) = x$ (Oussalah, 2003). The definition of t-norms does not imply any kind of continuity. Nevertheless, such a property is desirable from theoretical and practical perspectives. A t-norm is said to be continuous if it is continuous as a two-place function. T-norms can be classified as follows.

- A t-norm T is called Archimedean if it is continuous and $T(x,x) < x$, for $\forall x \in (0,1)$.
- An Archimedean t-norm T is called strict if it is strictly increasing in each variable $\forall x, y \in (0,1)$.
- An Archimedean t-norm T is called nilpotent if it is not strictly increasing in each variable $\forall x, y \in (0,1)$.

Three typical t-norms are minimum t-norm, product t-norm and Łukasiewicz t-norm, listed as follows (Noguera, Esteva, & Godo, 2010; Troiano, Rodriguez-Muniz, Marinaro, & Diaz, 2014; Yan et al., 2011; Yu et al., 2013): (1) Minimum t-norm, $T_{\min}(x,y) = \min(x,y)$; (2) Product t-norm, $T_P(x,y) = x \cdot y$; and (3) Łukasiewicz t-norm, $T_L(x,y) = \max(x+y-1, 0)$. These basic t-norms have some remarkable properties: the product t-norm T_P is one kind of strict Archimedean t-norms; the Łukasiewicz