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A two-stage acceptable hesitancy based goal programming framework to evaluating missing values of incomplete intuitionistic reciprocal preference relations

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ABSTRACT

There exist two main types of uncertainty for an intuitionistic reciprocal preference relation (IRPR). One is inconsistency among pairwise intuitionistic judgments, and the other is vagueness and incompleteness of judgments. It is important to capture and control uncertainty or hesitancy of the obtained results for evaluating missing values of incomplete IRPRs. In this paper, we put forward geometric consistency of incomplete IRPRs. A two-stage procedure comprising two goal programming models is developed to evaluate missing values of an incomplete IRPR. The first goal programming model is devised to minimize the inconsistency level of the resulting complete IRPR and control ratio-based hesitation indices of the evaluated intuitionistic judgments within a given acceptable threshold. The second goal programming model aims to seek the most fitting evaluation values in the sense of maintaining the inconsistency level derived by the first model. By applying the developed evaluation model and introducing a weighted AND-like representable Cross Ratio uninorm-based aggregation method, a procedure is then presented for solving group decision making problems with incomplete IRPRs. Three numerical examples including a comparative study are examined to illustrate the advantage and applicability of the developed framework.

1. Introduction

Analytic hierarchy process (AHP) introduced by Saaty (1980) is one of the most widely employed multi-criteria decision making (MCDM) methods. Its primary strength not only lies in its impartiality and using grade-based pairwise comparison to elicit decision-makers' preferences over alternatives, but also lies in its flexibility and validity to be combined with different techniques such as goal programming, fuzzy logic and quality function deployment (Saaty & Vargas, 2001; Vaidya & Kumar, 2006). In the classical AHP, decision-makers' judgments are represented by multiplicative preference relations (also called multiplicative comparison matrices). Another common representation adopts fuzzy logic based preference relations for dealing with decision-makers' judgments with vagueness and uncertainty. In recent years, fuzzy logic integrated with AHP has received increasing research attention and found huge applications for solving MCDM problems (Mardani, Jusoh, & Zavadskas, 2015). It is worth noting that fuzzy AHP has became the second most widely employed MCDM technique in an independent mode (just after the classical AHP) as per a recent survey by Mardani et al. (2015).

To deal with decision-makers' preferences with fuzziness and hesitancy, Atanassov's intuitionistic fuzzy sets (A-IFSs) (Atanassov, 1986) come out as a logical representation framework, where membership and non-membership degrees are separately characterized and hesitation margins are explicitly considered. A-IFSs have been widely applied in many areas such as decision modelling (Büyüközkan & Güleryüz, 2016; Chu, Liu, Wang, & Chin, 2016; Garg, 2016; He, Chen, Zhou, Liu, & Tao, 2014; Jiang, Xu, & Gao, 2015; Meng & Chen, 2016; Qi, Liang, & Zhang, 2015; Wang, Li, & Wang, 2009; Wang & Qian, 2007; Wu & Liu, 2013; Yue & Jia, 2015; İntepe, Bozdag, & Koc, 2013), pattern recognition (Boran & Akay, 2014; Chen & Chang, 2015), clustering analysis (Zhao, Xu, Liu, & Wang, 2012), classification and machine learning (Szmidt, Kacprzyk, & Bujnowski, 2014), to name a few. To better simulate decision-makers' pairwise judgments with imprecision and hesitancy, Xu (2007) employed the A-IFS theory to introduce the concept of intuitionistic reciprocal preference relations (IRPRs). An IRPR consists of two types of uncertainty. One is inconsistency among the judgments (Entani & Sugihara, 2012; Wang, 2015b), and the other is hesitancy and incompleteness of the given judgments.

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A number of researchers have paid their attention to the use of IRPRs in decision making under indeterminate environments (Xu & Liao, 2015). The feasible-region-based consistency definitions and priority derivation methods were proposed for IRPRs in (Behret, 2014; Gong, Li, Forrest, & Zhao, 2011; Gong, Li, Zhou, & Yao, 2009). The mathematic transitivity equation based additive or multiplicative consistency definitions of IRPRs and priority derivation methods were presented in (Liao & Xu, 2014a, 2014b; Wang, 2013, 2015a; Wang & Li, 2016; Wu & Chiclana, 2014; Xu, 2007; Xu, Cai, & Szmidt, 2011; Xu & Liao, 2014). Based on the multiplicative consistency (Xu et al., 2011), Xu and Xia (2014) developed an iterative algorithm to improve the consistency of an inconsistent IRPR. Xu and Liao (2014) further employed this multiplicative consistency to develop an intuitionistic fuzzy AHP method. Wang (2015a) pointed out that this multiplicative consistency is not robust to permutations of label names of alternatives, and defined geometric consistency of IRPRs from the viewpoint of the multiplicative transitivity of intuitionistic geometric indices of pairwise judgments. A logarithmic least square model was also established to obtain priority weights from IRPRs.

All element values are known for a complete pairwise comparison matrix. Given the reciprocity of a $n \times n$ pairwise comparison matrix, it implies that $n(n-1)/2$ judgments in the upper or lower triangular portion should be furnished by a decision-maker. Sometimes, however, the decision-maker is unwilling or unable to furnish his/her judgments over some pairs of alternatives because of insufficient information or limitations of human thinking. In this manner, an incomplete pairwise comparison matrix is furnished by the decision-maker (Dopazo & Ruiz-Tagle, 2011; Fedrizzi & Silvio, 2007; Harker, 1987; Herrera-Viedma, Alonso, Chiclana, & Herrera, 2007; Shiraishi, Obata, & Daigo, 1998).

An important research issue in decision making with incomplete pairwise comparison matrices is to deal with missing information. Different models have been developed to evaluate missing elements of incomplete crisp pairwise comparison matrices, such as geometric-mean-based methods (Carmone, Kara, & Zanakis, 1997; Harker, 1987), the characteristic polynomial based heuristic method (Shiraishi et al., 1998), the main-eigenvector based method and the logarithmic least square based method (Bozóki, Fülöp, & Rónyai, 2010), and the parametric goal programming method (Dopazo & Ruiz-Tagle, 2011). Recently, Wang (2015c) established a two-step goal programming model to deal with missing information for incomplete interval pairwise comparison matrices. For incomplete reciprocal preference relations, some consistency-based methods have been proposed to deal with missing information. For instance, based on additive consistency, Herrera-Viedma et al. (2007) put forward an iterative procedure to evaluate missing values and applied it to group decision making with incomplete reciprocal preference relations. Meng and Chen (2015) introduced an additively consistent index of reciprocal preference relations, and developed a goal programming model to determine missing values of incomplete reciprocal preference relations. Based on multiplicative consistency, Alonso et al. (2008) proposed an evaluation method, which was later extended to develop a consensus support system for solving group decision making problems with incomplete reciprocal preference relations (Alonso, Herrera-Viedma, Chiclana, & Herrera, 2010).

Xu et al. (2011) proposed the concept of incomplete IRPRs, where membership and non-membership degrees of an unknown judgment are both assumed to be missing. They put forward two algorithms for evaluating missing elements of incomplete IRPRs. Wu and Chiclana (2014) developed two formulae to evaluate membership and non-membership degrees of missing elements for incomplete IRPRs. However, the evaluation methods in Xu et al. (2011) and Wu and Chiclana (2014) are not always valid for incomplete IRPRs and are not robust with respect to permutations of label names of alternatives (see Example 2 in Section 5). In addition, membership and non-membership degrees of missing judgments are required to be entirely unknown in Xu et al. (2011) and Wu and Chiclana (2014). For some decision making cases, a decision-maker may supply the membership degree or non-membership degree of a judgment based on an optimistic or pessimistic attitude. The evaluation methods in Xu et al. (2011) and Wu and Chiclana (2014) cannot tackle such missing values.

The aim of this paper is to develop a goal programming framework for evaluating missing values of incomplete IRPRs, in which inconsistency and hesitancy of the complemented intuitionistic judgments are integrally addressed. We first define geometric consistency of incomplete IRPRs. This consistency reflects multiplicative transitivity of geometric indices of intuitionistic judgments in the complemented IRPR. A two-stage goal programming approach is then developed to evaluate missing values for incomplete IRPRs. The first stage establishes a goal programming model to minimize the inconsistency level of the complemented IRPRs, and ratio-based hesitation indices of the evaluated intuitionistic judgments are controlled by a given acceptable threshold. The second stage is a post-optimality analysis step analogous to that given by Siskos, Grigoroudis, and Matsatsinis (2005). In this post-optimality step, a goal programming model is developed to find the most fitting values from the optimal solutions derived by the previous stage such that the final complemented IRPR has minimum inconsistency and maximum hesitancy under the condition of ratio-based hesitation indices of the evaluated judgments being controlled within the acceptable threshold. Subsequently, we devise a weighted AND-like representable Cross Ratio uninorm based method to aggregate individual complemented IRPRs into a collective IRPR by directly using membership degrees of individual intuitionistic judgments. Finally, based on the aforementioned models, a procedure is developed for solving group decision making problems with incomplete IRPRs.

The organization of the paper is structured as follows. Section 2 recalls some concepts related to reciprocal preference relations and IRPRs with geometric consistency. A two-stage goal programming approach is developed to evaluate missing values of incomplete IRPRs in Section 3. Section 4 puts forward a procedure to solve group decision making problems with incomplete IRPRs. Three numerical examples including a comparative study are provided in Section 5. Section 6 concludes the paper.

2. Preliminaries

This section recalls some basic concepts to multiplicatively consistent reciprocal preference relations and geometrically consistent IRPRs.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a set of n alternatives. A pairwise comparison matrix $R = (r_{ij})_{n \times n}$ is called a reciprocal preference relation (Chiclana, Herrera-Viedma, Alonso, & Herrera, 2009; De Baets & De Meyer, 2005; Xia, Xu, & Chen, 2013), if R satisfies

$$0 \leqslant r_{ij} \leqslant 1, r_{ij} + r_{ji} = 1, r_{ii} = 0.5, \quad \forall i,j = 1,2,\ldots,n. \tag{2.1}$$

From the multiplicative viewpoint, the value r_{ij} in R can be interpreted as a cross ratio $\frac{r_{ij}/r_{ji}}{1+r_{ij}/r_{ji}} = r_{ij}$. If $0.5 < r_{ij} < 1$, we have $\frac{r_{ij}}{1-r_{ij}} = \frac{r_{ij}}{r_{ji}} > 1$, meaning that alternative x_i is superior to x_j with a ratio $\frac{r_{ij}}{r_{ji}}$. If $0 < r_{ij} < 0.5$, one has $0 < \frac{r_{ij}}{1-r_{ij}} = \frac{r_{ij}}{r_{ji}} < 1$, implying that alternative x_j is superior to x_i with a ratio $\frac{r_{ji}}{r_{ij}}$. If $r_{ij} = 0.5$, then $\frac{r_{ij}}{1-r_{ij}} = \frac{r_{ij}}{r_{ji}} = 1$, indicating the indifference between alternatives x_i and x_j.