Computers & Industrial Engineering 105 (2017) 348–361

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Extension of information axiom from ordinary to intuitionistic fuzzy sets: An application to search algorithm selection

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ARTICLE INFO

Article history:
Received 19 January 2016
Received in revised form 1 December 2016
Accepted 10 December 2016
Available online 20 December 2016

Keywords:
Intuitionistic fuzzy sets
Ordinary fuzzy sets
Information axiom
Uninformed search algorithms

ABSTRACT

Information axiom aims at minimizing the information content of the design in order to determine the best alternative satisfying the required design characteristics. In the literature, information axiom has been extended to fuzzy environment in order to capture impreciseness and vagueness in decision making problems. In the scope of this study, the ordinary fuzzy information axiom has been extended to intuitionistic fuzzy sets which attempt defining a fuzzy set with its membership, non-membership and hesitance. Thus, reflection of decision makers' hesitancy in information axiom has been better provided. In this paper, to illustrate the applicability of the proposed approach, triangular intuitionistic fuzzy information axiom is applied to uninformed search algorithm selection problem including unweighted, weighted, and multi-expert evaluation cases. The obtained results show the validity and efficiency of the proposed model. One-at-a-time sensitivity analysis is also applied to reveal the robustness of the given decision.

1. Introduction

Decision making is a mental process in order to select the best alternative among the possible alternatives. It is often hard to obtain a solution if there are multiple and conflicting criteria in decision making problems. There is a wide variety of multiple criteria decision making (MCDM) models in the literature in order to find an optimum solution for complex decision making problems. One of the MCDM models is multi criteria information axiom which is proposed to determine the best alternative among the alternatives that satisfy independence axiom of axiomatic design (Kahraman & Cebi, 2009). Multi criteria information axiom is based on design, system, and common ranges which enable defining lower and upper bounds of design targets and the distribution function of the system performance. This provides an important advantage since it does not force the decision maker to define a single numerical design target.

In the solution process of complex decision making problems, decision makers often face many problems with vague information. Since fuzzy set approaches are suitable to use when the modelling of human knowledge is necessary, fuzzy decision making methods have been developed for modelling impreciseness, vagueness, and uncertainty in MCDM problems (Kahraman, 2008). Therefore, it is the first time Kulak and Kahraman (2005a, 2005b) extended the information axiom under fuzzy environment and the new methodology has been used for the solution of decision making problems under fuzzy environment. Then, Kahraman and Cebi (2009) extended fuzzy information axiom to be used as a solution tool for all types of decision making problems. In addition to information axiom, Cebi and Kahraman (2010a) extended independence axiom under fuzzy environment. The developed fuzzy independence axiom helps designers (i) to take into account weak and strong relations in the design matrix, (ii) to select the best design among the designs created by different design parameters, and (iii) to define importance degrees of the design parameters.

Ordinary fuzzy sets have been recently extended to intuitionistic fuzzy sets, hesitant fuzzy sets, and type-2 fuzzy sets (Yager, 2009; Kahraman, Oztaysi, & Cevik Onar, 2016b). Intuitionistic fuzzy sets (IFS) consider membership and non-membership values of an element at the same time and the sum of these degrees must be at most equal to 1. In some cases, the available information is not sufficient to define the exact membership function for a certain design parameter since experts may have some hesitation in defining the membership function. Therefore, ordinary fuzzy information axiom can be extended to these types of fuzzy sets. Li (2013) extended ordinary fuzzy information axiom (IA) by using intuitionistic fuzzy sets. However, there are some deficiencies in the method proposed by Li (2013). The definition of information axiom with IFS has been given just only for non-normal fuzzy

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http://dx.doi.org/10.1016/j.cie.2016.12.012
0360-8352/

numbers. In case of normal fuzzy numbers, the membership degree of the most possible value should be equal to 1. In Li's (2013) paper, the membership degree is never equal to 1 since non-normal IFS are used. Hence, Li's approach does not work with normal intuitionistic fuzzy numbers (IFNs) because their scale given for functional requirements is composed of non-normal fuzzy numbers whereas these numbers must be normal fuzzy numbers. Since functional requirements (FRs) describe the desired states, FRs must be defined by using normal fuzzy sets. In Li's (2013) paper, this problem causes that the sum of membership degree and non-membership degree becomes larger than 1.0 in the aggregated evaluation of some alternatives such as [(0.174, 0.324, 0.087); **0.913**, **0.221**] and [(0.345, 0.648, 0.745); **0.913**, **0.470**] in their illustrative example.

In this paper, new definitions for ordinary fuzzy information axiom with IFS have been proposed to overcome the deficiencies given above. The proposed intuitionistic fuzzy information axiom (IFIA) lets both normal and non-normal fuzzy sets to be used. The rest of this paper is organized as follows; Section 2 presents the mathematical foundations of axiomatic design and theoretical improvements in the methodology. The preliminaries of intuitionistic fuzzy sets are given in Section 3. The proposed intuitionistic fuzzy information axiom approach has been introduced in Section 4. Section 5 presents an illustrative example for the developed method. Finally, concluding remarks are given in Section 6.

2. Traditional information axiom

2.1. Ordinary information axiom

The Information Axiom is utilized to select the best design among the designs that satisfy the Independence Axiom (Suh, 2001). It is represent with the information content, I_i, that is related to the probability of satisfying the given *FR*s. Information content for a given FR_i is defined as follows (Suh, 1990):

$$\mathrm{I}_i = \log_2\left(\frac{1}{p_i}\right) \tag{1}$$

where p_i is the probability of achieving the functional (design) requirement FR_i and log is either the logarithm in base 2 (with the unit of bits). I_i determines that the design with the highest probability of success is the best design. When all probabilities are equal to 1.0, the information content is zero, and conversely, the information required is infinite when one or more probabilities are equal to zero.

In any design situation, the probability of success is given by what designer wishes to achieve in terms of tolerance (i.e. design range) and what the system is capable of delivering (i.e. system range). "System pdf" in Fig. 1 represents the probability density function of the system and determines the system capability range (Kulak & Kahraman, 2005a, 2005b). As shown in this figure, the overlap between the designer-specified "design range" and the system capability range "system range" is the region where the acceptable solution exists. This overlapped region is called "common range".

Therefore, in the case of uniform probability distribution function p_i may be written as

$$\mathrm{p}_i = \left(\frac{\textit{Common range}}{\textit{System range}}\right) \tag{2}$$

Therefore, the information content is equal to

$$\mathrm{I}_i = \log_2\left(\frac{\textit{System range}}{\textit{Common range}}\right) \tag{3}$$

The probability of achieving FR_i in the design range may be expressed, if FR_i is a continuous random variable, as

$$\mathrm{p}_i = \int_{dr^1}^{dr^u} p_s(FR_i)\cdot dFR_i \tag{4}$$

where $p_s(FR_i)$ is the system pdf (probability density function) for FR_i. Eq. (4) gives the probability of success by integrating the system pdf over the entire design range. (i.e. the lower bound of design range, dr^1, to the upper bound of the design range, dr^u). In Fig. 2, system pdf is given as a normal distribution function. In this figure, the area of the common range (A_{cr}) is equal to the probability of success p_i (Suh, 1990).

Therefore, the information content is equal to

$$I = \log_2\left(\frac{1}{A_{cr}}\right) \tag{5}$$

2.2. Ordinary fuzzy information axiom approach

It is the first time multi-attribute fuzzy information axiom (FIA) was proposed by Kulak and Kahraman (2005a, 2005b) since the

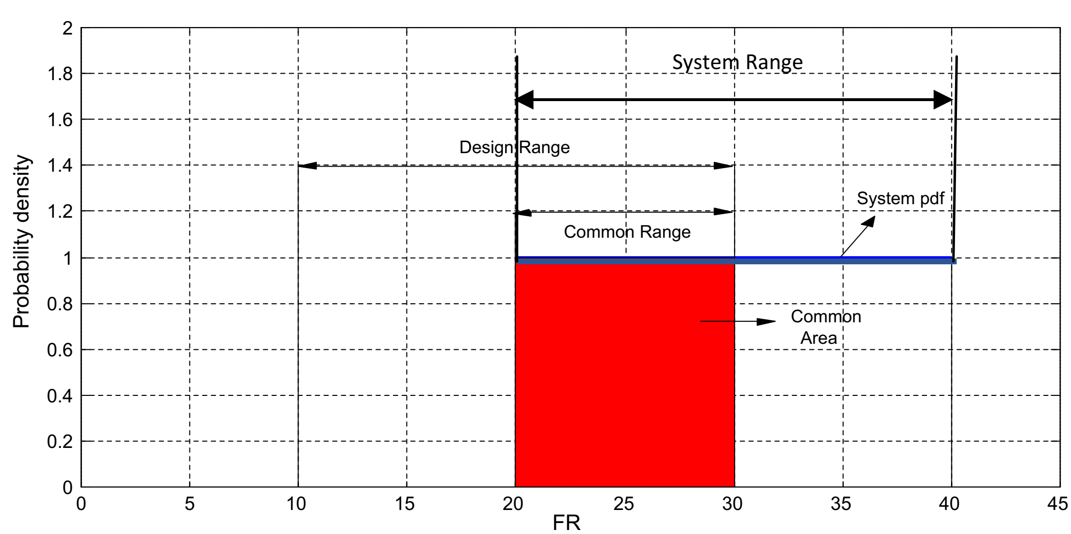

Fig. 1. Design range, system range, common range and probability density function of a *FR*.