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Fuzzy efficiency measures in data envelopment analysis using lexicographic multiobjective approach

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ABSTRACT

There is an extensive literature in data envelopment analysis (DEA) aimed at evaluating the relative efficiency of a set of decision-making units (DMUs). Conventional DEA models use definite and precise data while real-life problems often consist of some ambiguous and vague information, such as linguistic terms. Fuzzy sets theory can be effectively used to handle data ambiguity and vagueness in DEA problems. This paper proposes a novel fully fuzzified DEA (FFDEA) approach where, in addition to input and output data, all the variables are considered fuzzy, including the resulting efficiency scores. A lexicographic multi-objective linear programming (MOLP) approach is suggested to solve the fuzzy models proposed in this study. The contribution of this paper is fivefold: (1) both fuzzy Constant and Variable Returns to Scale models are considered to measure fuzzy efficiencies; (2) a classification scheme for DMUs, based on their fuzzy efficiencies, is defined with three categories; (3) fuzzy input and output targets are computed for improving the inefficient DMUs; (4) a super-efficiency FFDEA model is also formulated to rank the fuzzy efficient DMUs; and (5) the proposed approach is illustrated, and compared with existing methods, using a dataset from the literature.

1. Introduction

Data envelopment analysis (DEA), initially introduced by Charnes, Cooper, and Rhodes (1978), is a widely used mathematical programming technique for estimating the frontier production for peer decision making units (DMUs) with multiple inputs and multiple outputs. Charnes et al. (1978) model, commonly referred to as CCR model, assumed constant returns to scale (CRS). Banker, Charnes, and Cooper (1984) developed the so-called BCC model for evaluating the performance of units in the case of variable returns to scale (VRS). The units are assumed to operate homogenously under similar conditions. Based on the observed data and some preliminary assumptions, DEA is able to establish an empirical efficient frontier. If a DMU lies on the frontier, it is said to be *efficient*, otherwise it is said to be *inefficient*. Computing the distance to the efficient frontier (using some metric and a certain orientation) DEA provides the relative efficiency score, as well as a target for improving for each inefficient DMU. In practice, the efficiency score might be considered as a performance indicator for continuous improvement while the target informs about the amount (percentage) by which an inefficient DMU should decrease its inputs and/or increase its outputs to become efficient. Moreover, the reference set of efficient DMUs with which the target is constructed represents best practice models that act as benchmarks to the inefficient DMU.

In conventional DEA models, such as CCR and BCC, the observed input and output data of the DMUs are often not known precisely. That may not be always the case in the real world. Imprecise evaluations may be the result of unquantifiable, incomplete and non-obtainable information. Imprecise data representation with interval, ordinal, and ratio interval data was initially proposed by Cooper, Park, and Yu (1999, 2001a, 2001b), leading to so-called interval DEA (IDEA) to study the uncertainty in DEA. Numerous other researchers have also proposed and applied different DEA models with interval data (e.g. Despotis & Smirlis, 2002; Entani, Maeda, & Tanaka, 2002; Hatami-Marbini, Agrell, & Emrouznejad, 2014; Shokouhi, Hatami-Marbini, Tavana, & Saati, 2010; Shokouhi, Shahriari, Agrell, & Hatami-Marbini, 2014; Wang,

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Greatbanks, & Yang, 2005). However, decision makers often prefer using linguistic phrases and expressions such as "large" profit or "low" inventory in their communication, information that cannot be handled by IDEA. In general, observations are typically divided into quantitative and qualitative. Quantitative observed data are often exact, precise and specific values while qualitative data, such as "good", "better" and "very good", are often imprecise or vague values. The distances between qualitative data are not clear and it does not make sense to use the ordinal scaling to measure the preference linguistic terms that arise in natural language.

Fuzzy sets theory, initiated by Zadeh (1965), is a well-known tool to represent this type of data. Compared to traditional binary sets ("true" or "false", 0 or 1) fuzzy sets are based on the concept of "degree of membership", that ranges between zero and one. Natural language is not straightforwardly transformed into the absolute terms of 0 and 1. Fuzzy logic considers the membership values 0 and 1 as extreme cases but also considers possible intermediate membership values between 0 and 1. Hence, fuzzy sets have the capability of describing qualitative data as fuzzy numbers.

Numerous fuzzy sets-based methods have been proposed in DEA in the last two decades. Generally, the linear programming (LP) DEA models are converted to fuzzy LP (FLP) models when the input and/or output data are characterized by fuzzy numbers. The existing fuzzy DEA (FDEA) methods can be classified into six main categories, namely, the tolerance approach, the α-level based approach, the fuzzy ranking approach, the possibility approach, the fuzzy arithmetic, and the fuzzy random/type-2 (Emrouznejad, Tavana, & Hatami-Marbini, 2014; Hatami-Marbini, Emrouznejad, & Tavana, 2011).

The tolerance approach (e.g. Sengupta, 1992) was the first FDEA model that used the concept of fuzziness in DEA modeling by defining tolerance levels on constraint violations. The limitation behind the tolerance approach is related to the design of a DEA model with a fuzzy objective function and fuzzy constraints which may or may not be satisfied (Triantis & Girod, 1998).

The α-level approach is probably the most popular FDEA model in the literature. This approach generally tries to transform the FDEA model into a pair of parametric programs for each α-level. Kao and Liu (2000), one of the most cited studies in the α-level approach's category, used Chen and Klein (1997) method for ranking fuzzy numbers to convert the FDEA model to a pair of parametric mathematical programs for a given level of α. Saati, Memariani, and Jahanshahloo (2002) proposed a fuzzy CCR model as a possibilistic programming problem and changed it into an interval programming problem by means of the α-level based approach. Afterward, some fuzzy DEA-based extension has been done using Saati et al. (2002) method such as a four-phase fuzzy DEA framework based on the theory of displaced ideal (Hatami-Marbini, Saati, & Tavana, 2010) or a positive-normative use of fuzzy logic in a NATO enlargement application (Hatami-Marbini, Tavana, Agrell, & Saati, 2013). The α-level approach is also the one generally used in network DEA (e.g. Kao & Lin, 2012; Kao & Liu, 2011; Lozano, 2014a, 2014b).

The fuzzy ranking approach category is composed of FDEA models developed based on distinctive fuzzy ranking methods. Guo and Tanaka (2001) was the first to develop a fuzzy CCR model based on the fuzzy ranking approach. Different fuzzy ranking methods may lead to different efficiency assessments. Hatami-Marbini, Tavana, and Ebrahimi (2011) proposed a fully fuzzified CCR model to get the fuzzy efficiency of the DMUs where the input-output data as well as their weights are characterized by fuzzy numbers.

The "possibility approach" and the "credibility approach" to FDEA mainly stemmed from Lertworasirikul, Fang, Joines, and Nuttle (2003), which modeled the uncertainty in fuzzy objective function and fuzzy constraints with possibility measures from both optimistic and pessimistic viewpoints.

In the fuzzy arithmetic category, Wang, Luo, and Liang (2009) argued that a fuzzy fractional programming in the dual FDEA model cannot simply be transformed into a LP model using conventional methods. They therefore centered on the fuzzy fractional programming form of CCR model and transformed the multiplier formulation of the fuzzy CCR model into three LP models to obtain the fuzzy efficiency of the DMUs.

In the fuzzy random/type-2 category, Qin, Liu, Liu, and Wang (2009) presented a DEA model with type-2 fuzzy inputs and outputs solved in two steps. First, they exploited a reduction method for type-2 fuzzy variables based on the expected value of a fuzzy variable, and then they built a FDEA model with the obtained fuzzy variables. Qin and Liu (2010) developed a fuzzy random DEA (FRDEA) model where randomness and fuzziness exist simultaneously. The authors characterized the fuzzy random data with known possibility and probability distributions. Tavana, Khanjani Shiraz, Hatami-Marbini, Agrell, and Paryab (2012) also introduced three different FDEA models consisting of probability-possibility, probability-necessity and probability-credibility constraints in which input and output data entailed fuzziness and randomness at the same time.

In another category of fuzzy DEA models are those that make use of geometric properties. Thus, Biondi Neto, Meza, Gomes, and Bergiante (2011) developed a method to generate fuzzy efficient frontier by the use of interval DEA frontier when a single interval input or output presents a certain degree of uncertainty. The authors used a geometrical and algebraic approach to obtain a membership degree of each DMU in lieu of its efficiency score. In the same line, several researchers have defined the fuzzy version of the production possibility set (PPS) in which all production plans have different degrees of membership (Bagherzadeh Valami, Nojehdehi, Abianeh, & Zaeri, 2013; Raei Nojehdehi, Maleki Moghadam Abianeh, & Bagherzadeh Valami, 2012; Raei Nojehdehi, Maleki Moghadam, & Bagherzadeh Valami, 2011; Raei Nojehdehi, Valami, & Najafi, 2011). To do so, the authors first use a geometrical approach to acquire the membership function of fuzzy PPS and then transform the geometrical terms into the algebraic expression using some basic relationships of DEA models.

Apart from the tolerance approach, which exploits the fuzziness concept, FDEA models are generally represented as *FLP models* with fuzzy coefficients (i.e., fuzzy input-output data) and crisp decision variables. Since FDEA models take the form of FLP problems, the different FDEA approaches have been developed as different ways of solving the corresponding FLP models. In general, FLP problems can be classified into the following six categories to handle fuzzy data:

(1) FLP models when decision variables and the right-hand-side of the constraints are characterized by fuzzy numbers (e.g. Mahdavi-Amiri & Nasseri, 2007).
(2) FLP models when the coefficients of the decision variables in the objective function are characterized by fuzzy numbers (e.g. Wu, 2008).
(3) FLP models when the coefficients of the decision variables in the constraints and the right-hand-side of the constraints are characterized by fuzzy numbers (e.g. Liu, 2001).
(4) FLP models when the decision variables, the coefficients of the decision variables in the objective function and the right-hand-side of the constraints are characterized by fuzzy numbers (e.g. Ganesan & Veeramani, 2006).
(5) FLP models when the coefficients of the decision variables in the objective function, the coefficients of the decision variables in the constraints and the right-hand-side of the constraints are characterized by fuzzy numbers (e.g. Mahdavi-Amiri & Nasseri, 2006).