Computers & Industrial Engineering 105 (2017) 377–390

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

New unbalanced linguistic scale sets: The linguistic information representations and applications

Zhi Pei [a,b,*], Li Zheng [c]

[a] Department of Industrial Engineering, Zhejiang University of Technology, Hangzhou 310014, China
[b] Department of Industrial & Systems Engineering, North Carolina State University, Raleigh, United States
[c] Department of Industrial Engineering, Tsinghua University, Beijing 100084, China

ARTICLE INFO

Article history:
Received 10 March 2015
Received in revised form 11 October 2016
Accepted 17 December 2016
Available online 30 December 2016

Keywords:
Computing with words
Linguistic variables
Unbalanced linguistic cardinality
3-Tuple representation
Multi-attribute decision making

ABSTRACT

Traditionally, the linguistic variable based decision making mainly employs linguistic cardinalities with uniformly distributed scales. Within such scaling systems, the verbal information could be illustrated by multiple forms, such as the extended linguistic scale values and the 2-tuple linguistic representation. In general, the fundamental procedure to implement such methods involves the translation of the experts' verbal judgements into computable linguistic variables, the arithmetic operations of the linguistic variables, and the transformation of the derived outcomes back to the verbal terms. Meanwhile, still many experts prefer presenting their linguistic assessments on an unevenly distributed cardinality in contrast with the evenly distributed sets. In light of the mission "computing with words", it is essential to design a unified way to model the linguistic scale sets with both evenly and unevenly distributed cardinalities. Since the normal distribution serves as a decent tool to model the subjective judgements, in the present paper, a series of normal distribution based unbalanced linguistic scale sets are constructed. Besides, the area blocks under the distribution curve are applied to bridge the unbalanced linguistic variables and their relative coordinates. Furthermore, a novel 3-tuple format of unbalanced linguistic variable representation is proposed. The 3-tuple model is a generalized form of the linguistic information, and it could further degenerate to its 2-tuple linguistic counterpart. Finally, two numerical examples are included to demonstrate the validity of the proposed unbalanced linguistic variable based models and the 3-tuple linguistic information representation approach.

1. Introduction

In the book titled Analytic Hierarchy Process (Saaty, 1980), a detailed analysis is conducted concerning how to hierarchically scale the evaluation problems. In a classic AHP evaluation process, the assessments of each alternative with respect to different attributes or criteria are either real or integer numbers. During the evaluation process, three widely known scale sets are employed, which are $\{1,2,3\}, \{1,2,3,4,5\}$, and $\{1,2,3,\ldots,9\}$. Among all these three scale sets, the smallest element constantly represents the worst alternative, and vice versa. According to the expertise in the specific fields and personal experiences of the decision maker, the cardinality mostly deemed fit from the preselected scale set will be assigned to the inspected alternative. Even though the past research works relied heavily on the aforementioned three scale sets, still many other types of scales sets are developed and utilized nowadays. One major rule to develop a new scale set is to match its extent of precision to the needs of the decision making process. In other words, the denser the cardinalities pertained in one specific scale set, the more delicate assessment the alternatives could receive. However, denser cardinalities might also incur unnecessary decision making burden, such as the difficulty in distinguishing the close scale values, hence finer granularity within one scale set will demand higher level of proficiency in judgement. For convenience, in the present paper, if a scale set contains *n* scale values, then the set is referred to as an *n*-scale set. Recent studies have revealed, in general, that the 9-scale set seems more applicable and reasonable compared with the other scale sets. Although the 5-scale set and 3-scale set could also handle decision making tasks, under certain circumstances, they simply cannot provide sufficient granularity to distinguish different alternatives.

Since many experts express their opinions with words, such as "good", "bad", or "fair", besides the cardinalities in the form of numbers, the ambiguity and fuzziness included within words

* Corresponding author at: Department of Industrial Engineering, Zhejiang University of Technology, Hangzhou 310014, China.
E-mail address: peizhi82@163.com (Z. Pei).

http://dx.doi.org/10.1016/j.cie.2016.12.024
0360-8352/

could be better modeled via linguistic based formulations. Among these formulations there are three fundamental approaches (Herrera & Herrera-Viedma, 2000), the semantic model, the symbolic model, and the linguistic 2-tuples model (Martinez & Herrera, 2012). In these models, the cardinalities of the linguistic terms substitute the traditional forms of number-based scale sets. In decision making problems with linguistic variables, the linguistic information is denoted as S_α. To differentiate the linguistic scale sets listed in the present paper from the traditional linguistic information representations, hereafter the linguistic scale set with n cardinalities is denoted as n-LingScale set.

Ever since its introduction, the linguistic formed judgements have been widely applied to various practical fields (Xu, 2006, Xu, 2009). In recent years, the applications of linguistic terms in industrial cases have gained more attention from the research community. A fuzzy linguistic method is employed to obtain quality assessment for electricity network infrastructure (Celotto, Loia, & Senatore, 2015). A linguistic multi-criteria decision making model is utilized to select the contracted data transmission link between two geographic points in the city for a telecommunication enterprise (Cid-Lopez, Hornos, & Carrasco, 2016). The 2-tuple linguistic method could also be applied to construct the house of quality during new product design phase (Ko, 2015). A symmetrical interval intuitionistic linguistic operator is proposed to deal with an engine performance problem (Meng & Chen, 2016). The subjective and objective linguistic information has been combined together to assist the decision making with regard to the EU regulation creation (Merigo, Palacios-Marques, & Zeng, 2016). A linguistic preference based group decision making approach is introduced (Tao, Liu, & Chen, 2015), and it is applied to solve a vehicle noise quality evaluation problem. A group multi-granularity linguistic method is employed to deal with a crucial stage of quality function deployment (QFD) (Wang, Fung, & Li, 2016). The innovation capability could also be evaluated based on linguistic variables (Yang, Zhang, & Ding, 2015). A generalized asymmetric linguistic term set has been introduced (Zhou & Xu, 2016), and applied to solve investment selection problem with different risk appetites. Besides, the linguistic truth-valued intuitionistic fuzzy reasoning is applied to tackle safety management problem in marine engineering (Zou, Wen, & Wang, 2016). Last but not least, the non-formatted text has been transformed into multi-granularity linguistic information to present a novel project evaluation method (Zhu, Wang, & Chen, 2015).

While using the extension principle to quantify different alternatives, in order to match the linguistic variables to each assessment in the information aggregation operation, in literature, fuzzy triangular numbers and fuzzy trapezoidal numbers are implemented to represent the linguistic variables. These special fuzzy numbers are then weighted and aggregated to derive the comprehensive assessments of the alternatives. Furthermore, under certain circumstances, more sophisticated ranking methods could be applied. Take the fuzzy TOPSIS method for example, the aggregated fuzzy quantities are compared with the positive/negative ideal alternative, and then the ratio of the distances could be calculated to facilitate in the final ranking. It is recognized that throughout the evaluation process, the input information used by far is merely the experts' verbal judgements, while the output information is the fuzzy number based evaluation toward one specific alternative. Therefore the extension principle based computation acts as a medium, the overall procedure of the linguistic variable based decision making fulfills the mission of "computation with words" (Zadeh, 1965).

Besides the transformation of the linguistic variables into the fuzzy quantities, in order to properly represent the linguistic information, some approaches may even skip the transformation process (Herrera, Lopez, & Rodriguez, 2002), and they are conducted directly based on linguistic cardinalities. Commonly, the weighted aggregation of linguistic variables may yield non-integer cardinalities. Since these fuzzy variables with fractional cardinalities cannot be denoted by the existing integer linguistic scales, the aggregated results are rounded to their nearest integer linguistic cardinalities instead.

The two aforementioned linguistic information based decision making methods both include approximation of the decision maker's original verbal assessments. This sort of approximations will lead to information leakage, and possibly weaken the integrity of the intended evaluation. In order to overcome this weakness, a 2-tuple form of expression is proposed to symbolize the linguistic evaluations, and especially the non-integer cardinalities (Herrera & Martinez, 2000). To clarify, the so-called 2-tuple representation is denoted as (S_i, α), in which S_i indicates the ith cardinality of the linguistic scale set m-LingScale. Without loss of generality, it is presumed that the comprehensive evaluation term is closest to, among all the other linguistic cardinalities, S_i. Then the distance between the comprehensive evaluation term and S_i could be employed to pinpoint the exact position of the linguistic assessment. The parameter α stands for the fractional part in the half open interval $[-0.5, 0.5)$. Intuitively, the negative value of the parameter α indicates that the evaluation outcome is on the left side of S_i, and vice versa. When $\alpha = 0$, the 2-tuple degenerates to a classic linguistic variable, which coincides with the cardinality S_i.

As it is pointed out by Herrera et al., linguistic information aggregation process in the form of 2-tuples incurs no information loss whatsoever, and it can distinguish any pair of different linguistic evaluations (Herrera & Martinez, 2000). More recently, a more direct information aggregation method based on linguistic variables has been introduced (Xu, 2004). This approach extends the traditional discrete linguistic cardinalities to continuous linguistic scale set, in which each coordinate on the scale axis represents a specific linguistic cardinality. Apparently, countless such linguistic cardinalities exist between two adjacent integer based linguistic cardinalities. Take S_1 and S_2 for instance, in between them the cardinalities with fractional parts include $S_{1.1}, S_{1.2}, S_{1.3}$, and so on. All these three listed non-integer cardinalities have portrayed the decision maker's opinion, which is somewhere between the linguistic cardinalities S_1 and S_2, leaning toward the position of S_1.

Furthermore, in the practical decision making process, since the discrete linguistic cardinalities are more commonly accepted, the continuous linguistic cardinalities stemming from their discrete counterparts could only serve as a tool of virtual scaling to assist in actual ranking. In real fuzzy decision making problems with linguistic variables, it is very important to select a series of proper linguistic cardinalities. To the best of our knowledge, most of the existing literature employs the linguistic cardinality with uniformly distributed scale pattern, which means that the granularity between any two adjacent linguistic cardinalities is the same (Xu, 2009; Xu & Wang, 2015). Nevertheless, in reality, when it comes to the subjective nature presented by the decision maker, the granularity between each pair of cardinalities may vary. In a more recent paper(Herrera, Herrera-Viedma, & Martinez, 2008), it mentioned that for unbalanced linguistic cardinalities, the scale values could be mapped to a series of hierarchically structured uniformly distributed scale sets (Cid-Lopez et al., 2016; Herrera, Herrera-Viedma, & Martinez, 2000; Herrera & Martinez, 2001; Jiang, Fan, & Ma, 2008). Then the evenly distributed scale values are employed to substitute the original unbalanced ones in information aggregation operations. Finally, the computation results are transformed back into the linguistic terms under the unbalanced cardinality via a reverse process. Throughout the procedures of back and forth transformation, the linguistic variables are presented in the form of 2-tuples and real numbers. Unfortunately, the articles concerning the definitions and computation methods