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An analysis of formulations for the capacitated lot sizing problem with setup crossover

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ABSTRACT

The lot sizing problem with setup crossover is an extension of the standard big bucket capacitated lot sizing problem (CLSP). The general idea is that the first setup operation of each planning period can already start in the previous period, if not all the capacity is used in that previous period. This provides more flexibility in the planning and increases the possibility of finding feasible and better solutions compared to the standard assumption. Two different formulations have been presented in the literature to model a setup crossover. Since these formulations have not been compared directly to each other, we present a computational study to determine which is the best formulation. Furthermore, we explore ideas indicating that in one of the formulations from the literature it is not necessary to impose binary conditions on the crossover variables and we propose symmetry breaking constraints for both formulations from the literature. Finally, we quantify the value of this type of flexibility in a computational experiment and analyze which factors influence this value.

1. Introduction

The research on dynamic lot sizing in discrete time started over 50 years ago with the seminal papers of Wagner and Whitin (1958) and Manne (1958). Over the past decades, there has been an increasing interest in the application of these models, and researchers have been able to incorporate more and more real world features into lot sizing problems.

The lot sizing problem is a production optimization problem which involves determining how many items to produce in each period in order to meet the demand for these items. The resulting production plan should minimize the sum of the setup, production and inventory holding costs. The problem considered in this work is the single stage, single machine, multi-product, big time bucket lot sizing problem with setup times. Several different products can be produced in the same time period on the same machine. A setup must be done for each type of product that is produced in a specific period. In the standard version of this problem the setup for the first product type produced in a period starts at the beginning of that period (Trigeiro, Thomas, & McClain, 1989). In this paper we study an extension of this lot sizing problem that includes the possibility of a setup crossover. The idea is that in certain cases setup operations can be interrupted at the end of a period and resumed at the beginning of the next period, in other words, the setups can span over two periods. This implies that the first setup in period t can already start at the end of period $t-1$ if there is some capacity left, and continue at the beginning of period t (Menezes, Clark, & Almada-Lobo, 2010). This flexibility can result in more efficient solutions compared to the standard assumption (where the setup time is restricted to be contained within the period) since we free up capacity in period t by (partially) moving the setup of the first product to the previous period. In the big bucket models, the setup times are smaller than the capacity limit.

Typically, the setups include machine adjustments, calibration, inspection and cleaning activities that are required before switching over the resource to produce another product. Quite often, setup operations can be interrupted at the end of a period (e.g. just before the weekend break) and resumed at the beginning of the next one (e.g. just after the weekend break). In other cases, the operation is run continuously, and there is no period of interruption between the end of one period and the start of the next one. In both cases, the setup can be split between two periods. We give some examples. In the beverage industry, the setup of the beverage production line consists in preparing the syrup in tanks and distribute it to parallel bottling machines. In some cases this process can be interrupted and resumed the next period. In the fabrication of steel components, different molds are needed to produce differ-

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ent products. The setup consists of changing the mold and in some cases this process can be interrupted and resumed without any problem.

It is important to note the differences between the concepts of setup crossover and setup carryover. While with setup crossover the setups can span over two periods, the setup carryover allows a setup state to be maintained from one period to the next one. In other words, if we finish a period t producing a particular item i it is possible to start producing the item i in period $t+1$ without performing a new setup for this item.

Although setup crossover is a natural extension of the standard assumption, just a few studies have considered it, due to the difficulty in dealing with the underlying problems (Mohan, Gopalakrishnan, Marathe, & Rajan, 2012; Belo-Filho, Almada-Lobo, & Toledo, 2014). All the studies that handle setup crossovers in their formulations have added extra binary variables to the formulations indicating if there is a setup crossover in a period or not, which increases the difficulty of the formulations.

The aim of this paper is: (1) to compare the two formulations proposed in the literature to determine which formulation is the best; (2) to propose new constraints to break the symmetry which is present in the formulations from the literature; (3) to prove that in one of the formulations from the literature we do not need binary conditions on the crossover variables; (4) to analyze the impact of the proposed adaptations of these formulations (i.e. no binary variables and symmetry breaking constraints) in computational experiments, and (5) to determine the value of the flexibility provided by the setup crossover and analyze the factors that have an impact on this value.

We also have explored other ideas to avoid the necessity of defining new extra binary variables to model the setup crossover. Two new formulations were proposed and can be found in a technical report (Fiorotto, Jans, & de Araujo, 2014) which includes some theoretical and computational results. These two formulations present more restricted models, and hence provide only an upper bound on the optimal solution for the model with setup crossover. The computational experiments indicated that these two restricted formulations without extra binary variables for the setup crossover actually take substantially more time to be solved compared to the best formulation for the setup crossover. Therefore, these two restricted formulations are not included in this paper.

The paper is organized as follows. In Section 2, we provide a literature review on lot sizing problems with setup crossover. Section 3 presents the formulations from the literature along with the new proposed formulations including some theoretical results for the formulations. Section 4 describes the computational results and analyses and finally in Section 5, we present our conclusions.

2. Literature review

There is a vast amount of literature on big-bucket capacitated lot sizing problems (CLSP) with setup times, where setup times have to be contained completely within one period (Trigeiro et al., 1989). These models have been extended to deal with various industrial issues (see Jans & Degraeve (2008) for an overview), including setup carryover and setup crossover.

Several papers analyze the extension with setup carryover. Sox and Gao (1999) propose two formulations for the CLSP with setup carryover. The first one extends the formulation proposed by Trigeiro et al. (1989) and the second one uses the shortest path reformulation and the ideas proposed by Eppen and Martin (1987). Suerie and Stadtler (2003) propose a formulation for the CLSP with setup carryover based on the simple plant location formulation (Krarup & Bilde, 1977) and their computational tests have shown that this formulation is better than the formulations proposed by Sox and Gao (1999). Gopalakrishnan, Ding, Bourjolly, and Mohan (2001) develop a tabu search heuristic to solve the CLSP with setup carryover and using the data sets from Trigeiro et al. (1989) they compute the effectiveness of the setup carryover. Their results indicate an 8% reduction in total cost on average through setup carryover compared with the standard CLSP.

Regarding the problem with setup crossover for the small bucket problem, Suerie (2006) studies the lot sizing and scheduling problem and proposes two formulations that correctly handle setup crossovers which allow "long" setup times (i.e. setup times can be bigger than the capacity in one period). The author compares his results with the results found by the standard lot sizing and scheduling problem and concludes that the proposed formulations remove infeasibility and produce improved solutions in certain cases.

For the big bucket problem, Sung and Maravelias (2008) present a mixed-integer programming formulation for the capacitated lot sizing problem allowing setup carryover and crossover (CLSP-SCC). The authors consider sequence independent setups, non-uniform time periods and long setup times. They show in a detailed way how to deal with the boundary of the periods using setup crossover with the assumption that the setup cost is accounted for at the beginning of the setup. Finally they discuss how their formulation can be extended for problems with idle time, parallel units, families of products, backlog and lost sales.

Menezes et al. (2010) propose a formulation for the CLSP-SCC considering sequence-dependent and non-triangular setups, allowing subtours and enforcing minimum lot sizing. They propose two lemmas to demonstrate that their formulation is more efficient than the classical lot sizing and scheduling problem. Moreover, they present an example that shows the improvement of the solutions allowing setup crossover compared to the classical formulation.

Kopanos, Puigjaner, and Maravelias (2011) develop a formulation for the CLSP-SCC with backlog where the items are classified into families. The approach considers that the setups are family sequence-dependent, and sequence-independent for items belonging to the same family. The formulation is tested for a complex real world problem in the continuous bottling stage of a beer production facility and it finds good solutions for problems with hundreds of items.

Mohan et al. (2012) include the possibility of setup crossover for the formulation proposed by Suerie and Stadtler (2003) that handles the problem with setup carryover and compare the improvement obtained by adding the crossover in the formulation with setup carryover. They conclude that in nine out of fifteen problem instances tested, their formulation yielded better solutions or removed infeasibility.

Camargo, Toledo, and Almada-Lobo (2012) propose three formulations for the two-stage lot sizing and scheduling problem and one of these considers setup crossover, which is achieved by a continuous-time representation. From the computational results, they conclude that despite delivering the worst performance in terms of CPU times, the formulation with setup crossover is the most flexible of the three to incorporate setup-related features.

Belo-Filho et al. (2014) consider the problem CLSP-SCC with backlog. They propose two formulations for the problem, the first one is built on top of the formulation of Sung and Maravelias (2008) and the second one proposes a time index disaggregation, defining the start and the completion time periods of the setup operation. They show the relationship between the proposed formulations and compare their formulations with the formulation proposed by Sung and Maravelias (2008). Finally they point out that setup crossover is an important modeling feature in case setup times consume a considerable part of the period capacity.