Computers & Industrial Engineering 107 (2017) 313–326

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A two-stage stochastic programming model for periodic replenishment control system under demand uncertainty

P.S.A. Cunha [a], F.M.P. Raupp [b], F. Oliveira [a,*]

[a] Departamento de Engenharia Industrial, Pontifícia Universidade Católica do Rio de Janeiro, Rua Marquês de São Vicente, 225, Gávea, Rio de Janeiro, RJ 22451-900, Brazil
[b] Laboratório Nacional de Computação Científica, Av. Getúlio Vargas, 333 – Quitandinha, Petrópolis, RJ 25651-075, Brazil

ARTICLE INFO

Article history:
Received 25 February 2015
Received in revised form 5 December 2016
Accepted 21 March 2017
Available online 23 March 2017

Keywords:
Inventory policy
Replenishment control system
Periodic review
Uncertain demand
Stochastic programming

ABSTRACT

The inventory control system consists of defining strategies to manage inventory replenishment such that costs involved in ordering, holding, and meeting the demands are optimized. Although several methods for inventory management are proposed in the literature, the assumptions made necessary can hinder their applicability in practice. Motivated by this fact, we propose a methodology for determining the optimal parameters of an inventory control system for single-item one-echelon supply chains using two-stage stochastic programming, considering periodic review and uncertain demand. The proposed approach is flexible enough to consider backlogs or lost sales cases without limitations on the number of outstanding orders and allows the consideration of uncertainties in a more comprehensive manner when compared to currently available methods. The optimal review periodicity and the inventory target level are determined using a nonlinear mixed-integer programming model, which takes into account the uncertain nature of the item demand levels through a finite set of scenarios. We present how this model can be reformulated as a deterministic equivalent mixed-integer linear programing model and how the Sample Average Approximation method can be used to incorporate the uncertainty into it. To validate the proposed approach, we perform a simulation-based sensitivity analysis in the neighborhood of the solutions obtained. We also compare them with those obtained applying the Hadley-Whitin method, considering the premises that are necessary for the later. Further, we present results from a case in which the stochastic demand is not stationary. The results obtained provide strong evidence of the potential of the proposed approach.

1. Introduction

Inventory management issues in supply chains (SCs) pervade the decision-making process of many companies. Typically, decisions regarding the coordination of existing demands with supply of products and inputs of materials in time and space dimensions must be made such that specified costs and service levels goals are met. The key questions that inventory managers have to answer are when to order, how much to order, and how much to keep in safety stocks (Namit & Chen, 1999).

Inventory can be either continuously or periodically reviewed at regular intervals of length R (Silver et al., 1998). The order quantity can be fixed (order of size Q) or variable (order size is such that the inventory position is increased to a target level S). When no fixed order cost is charged, orders are simply placed at every reordering moment. However, the existence of ordering costs requires that the cost of placing an order is considered in the decision of whether to place an order or not. The literature contains several inventory policy proposals for single-echelon SCs that aim to provide decision guidelines in terms of order placement for inventory management, often referred to as inventory control systems. Among them, the four most common inventory control systems are the continuous review (s, Q) and (s, S) systems, and the periodic review (R, S) and (R, s, S) systems, where s denotes the ordering point (Hadley & Whitin, 1963; Silver, Peterson, & Pyke, 1998; Zipkin, 2000).

Replenishment control systems with periodic review are widely used, as they require less transactional effort, involve easier planning for calculations of workload needs, facilitate customer service and receiving from suppliers, allow better replenishment coordination, especially when involving multiple items, and generate more stability to the SC. Moreover, when dealing with stationary demand in a single-item single-echelon SC, a periodic review policy returns the best results in terms of total costs, and, in the case of a multi-echelon SC, it has the advantage of being simpler to implement (Federgruen & Zipkin, 1984).

* Corresponding author.
E-mail address: fabricio.oliveira@puc-rio.br (F. Oliveira).

http://dx.doi.org/10.1016/j.cie.2017.03.025
0360-8352/

Ideally, an inventory control model should be able to consider most of the characteristics of its corresponding SC, including backlogs and lost sales. However, according to Bijvank and Vis (2011), there are few inventory models considering lost sales in the literature. Even though it is often more appropriate to model customer's behavior with lost sales compared to backlog-based models, inventory models that include lost sales are typically more complex, since the consideration of more than a few outstanding orders can lead to computationally intractable models. Moreover, when a lost-sales based model is approximated by a backlog-based model, costs deviations can be non-negligible (Zipkin, 2008a).

Most of the research on SC assumes that its operational characteristics are deterministic. However, some parameters such as customer demand, prices, and resource capacities, can be subject to uncertainty, which can generate undesired impacts on the ability to serve customers, and ultimately reaching SC goals. The importance of uncertainty has lead several authors to treat the distribution of raw materials and products during the planning of SCs - at a tactical level - and the location of facilities - at a strategic level - as stochastic problems (Santoso, Ahmed, Goetschalckx, & Shapiro, 2005). While the deterministic case is well developed, the existing scientific literature addressing inventory control systems under uncertainty considers probabilistic parameters in an approximated fashion and require restrictive assumptions.

The objective of this paper is to present the use of two-stage stochastic programming to support decision making related to the management of a single-item inventory system in a single-echelon SC, whose demand levels are uncertain over a finite time horizon. To this end, we propose a model that is capable to define the optimal parameters of (R, S) inventory control systems in terms of costs, assuming constant periodic review and variable order quantity. In the model, the relevant costs are the ordering, carrying, and shortage costs, which are considered deterministic but possibly changing along the planning horizon, and the demand fulfilment being either possibly postponed (referred hereinafter as backlog case) or not fulfilled (i.e., lost sales case). In addition, we present how the sampling-based approach known as Sample Average Approximation (SAA) can be used to generate finite discrete scenario samples to represent the item demand and obtain arbitrarily good solutions for the problem. Moreover, we performed computational experiments considering instances that were generated to illustrate the potential of the proposed methodology.

It is worth mentioning that the application of the proposed methodology is not limited to the premises imposed by existing stochastic methods in the literature. In addition to a simulation-based sensitivity analysis performed to assess the solutions obtained using the proposed approach, we choose as reference the stochastic Hadley-Whitin (HW) method to compare the numerical results, since it also addresses a periodic review inventory system, it is straightforward to implement, and it is known to be capable of providing nearly optimal solutions for the lost sales case in a given range of values of holding and shortage costs (please refer to Hadley and Whitin (1963) for a detailed discussion on this regard). Moreover, to the best of our knowledge, the proposed approach was not found in any other research work currently available in the literature, which aims to determine the optimal parameters values of a periodic review inventory policy in the case considering lost sales (although both cases – lost sales and backlog – are possible to be considered in the proposed framework, we focus on the lost sales case due to its scarcer literature body, as recognized in Bijvank and Vis (2011)).

The contributions offered by this research can be summarized as follows. First, the use of mathematical programming to model the problem of defining optimal inventory control policies considering periodic review provides a more flexible setting, as it allows the consideration of both backlog and lost sales (without limitation on the number of outstanding orders) cases in a fairly straightforward manner and without compromising computational tractability, a recurring issue faced by alternative approaches in the literature. In particular, the proposed model is capable to consider the lost sales case without limitations on the number of outstanding orders, as well as lead times that are independent of the review periodicity, which are, as will be discussed further in the following sections, limiting characteristics often inherent to the models available in the literature. In addition, as it does not require restrictive assumptions such as time independence, normality, and fixed costs along the planning horizon, the proposed approach can thus be applied to a wider range of problems.

Second, we propose a novel two-stage stochastic programming model that can be reformulated into an equivalent deterministic mixed-integer linear programming (MILP) model. The model has the objective of defining optimal strategies of a replenishment control system with constant periodic review and variable order quantities in regards to order, holding and shortage costs. The optimal parameters values (R, S) are obtained through an approach based on SAA, which allows stochastic phenomena to be considered in a more adherent manner, without relying on any specific scenario generation method to obtain a discrete representation of the random phenomena.

The paper is structured as follows. Section 2 introduces the literature review related to the problem of interest and Section 3 presents the description of the addressed problem. The Hadley-Whitin method and its mathematical formulation are briefly presented in Section 4. The proposed methodology for determining optimal parameters for the periodic review (R, S) system is presented in Section 5, along with the SAA approach. Numerical results are presented for several instances in Section 6, and conclusions are provided in Section 7.

2. Literature review

In the context of inventory control systems, most of the literature focuses on strategic decisions of single-echelon logistics networks. Recent studies, such as Gupta and Maranas (2000), Santoso et al. (2005), Oliveira and Hamacher (2012), and Oliveira, Gupta, Hamacher, and Grossmann (2013) address multi-echelon problems through stochastic programming. Despite considering inventory management and SC design in a joint manner, those studies do not directly address inventory policies. However, Daskin, Coullard, and Shen (2002), Shen, Coullard, and Daskin (2003), and You and Grossmann (2008) address multi-echelon SC design and inventory policy without using the stochastic programming technique. More recently, Fattahi, Mahootchi, Moattar Husseini, Keyvanshokooh, and Alborzi (2015) propose an inventory policy for a two-echelon logistics network based on the (s, S) continuous review system, considering a single item with uncertain demand using stochastic programming.

Under a production planning point of view, Glock (2012) and Glock, Grosse, and Ries (2014) provide reviews of lot-sizing problems classifying them according to two basic dimensions, namely, how the nature of the product's demand is modeled, and what cost parameters are considered in the model. Regarding the first dimension, models differ by how parameters vary in time (stationary or dynamic parameters, respectively) and whether the uncertainty is considered or not (deterministic or stochastic models, respectively). As for the second dimension, the models can be classified as "classic" or "extended". The classic models depict approaches aimed at defining the optimal amounts of production, orders, and shipments, and hence they consider in its formulation costs of ordering, set up, carrying inventory and shipping. These models can be seen as variants of the Economic Order Quantity (EOQ) basic