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Technical Note

# A note on "The EOQ repair and waste disposal model with switching costs" ☆

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ABSTRACT

The EOQ repair and waste disposal problem was studied first by Richter (1997) and was extended by Saadany and Jaber (2008) to the problem of minimizing the total cost of production, remanufacturing and inventory while incorporating additional switching costs. However, in their paper the authors did not provide a complete solution to this complex problem. We provide the solution in this note.


## 1. Introduction

In recent years, reverse logistics has been receiving increasing attention from academia and industry. The integration of forward and reverse supply chains resulted in the origination of the concept of a closed-loop supply chain. The whole chain can be designed in such a way that it can service both forward and reverse processes efficiently. The subject of this note is deterministic inventory models with constant demand and return on the base of EOQ (economic order quantity).

A latest actual survey of mathematical inventory models for reverse logistics can be found in Bazan, Jaber, and Zanoni (2016). An extensive survey of research related to quantitative modeling for inventory and production planning in a closed-loop supply chain was provided by Akçalı and Çetinkaya (2011). More recent papers which have considered inventory and production planning models with constant demand and return on the base of EOQ Saadany, Jaber, and Bonney (2013), Hasanov, Jaber, and Zolfaghari (2012), Guo and Ya (2015), Bazan, Jaber, and Saadany (2015), and Saadany, Jaber, and Bonney (2013).

In the paper of Saadany and Jaber (2008) the extended EOQ production, repair and waste disposal model of Richter (1996) was modified to show that ignoring the first time interval results in an unnecessary residual inventory and consequently an over estimation of the holding costs. They also introduced switching costs in order to take into account production losses, deterioration in quality or additional labor. When shifting from producing (performing) one product (job) to another in the same facility, the facility may incur additional costs, referred to as switching costs, when alternating between production and repair runs. The special case of even numbers $m$ and $n$ was studied and conditions were provided to decide which of two policies $P(m,n)$ and $P(\frac{m}{2},\frac{n}{2})$ is preferable, but a general optimal policy for the problem was not presented. In our study we will provide a general optimal solution for the model. The proposed approach can be further implemented to solve a whole class of deterministic inventory models with constant demand and return rates. More detailed analysis can be found in working paper (Kozlovskaya, Pakhomova, & Richter, 2015).

## 2. Assumptions and notations

### 2.1. Assumptions

This note assumes: (1) infinite manufacturing and recovery rates; (2) repaired items are as good as new; (3) demand is known, constant and independent; (4) the lead time is zero; (5) a single product case; (6) no shortages are allowed; (7) unlimited storage capacity is available; and (8) an infinite planning horizon.

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### 2.2. Notations

$T$ – length of a manufacturing and repairing time interval (units of time), where $T > 0$
$T_1$ – length of the first manufacturing time interval (units of time), where $T_1 < T$ and $T_1 > 0$
$n$ – number of newly manufactured batches in an interval of length $T$
$m$ – number of repaired batches in an interval of length $T$
$d$ – demand rate (units per unit of time)
$h$ – holding cost per unit per unit of time for shop 1
$u$ – holding cost per unit per unit of time for shop 2
$\alpha$ – waste disposal rate, where $0 < \alpha < 1$
$\beta$ – repair rate of used items, where $\alpha + \beta = 1$ and $0 < \beta < 1$
$x$ – batch size for interval $T$, which includes $n$ newly manufactured and $m$ repaired batches; $x = dT$
$r$ – repair setup cost per batch
$s$ – manufacturing setup cost per batch
$r_1$ – setup and switching costs of the first repair run
$s_1$ – setup and switching costs of the first production run, denoted by $r_1 = r + \textit{switching cost}$ from production to repair, and $s_1 = s + \textit{switching cost}$ from repair to production.

## 3. Formulation of the model and its analysis

Richter (1996) introduced an EOQ repair and waste disposal model. A first shop is providing a homogeneous product used by a second shop at a constant demand rate of $d$ items per time unit. The first shop is manufacturing new products and it is also repairing products used by a second shop, which are then regarded as being as good as new. The products are employed by a second shop and collected there according to a repair rate $\beta$. The other products are immediately disposed of as waste according to the waste disposal rate $\alpha = 1 - \beta$. At the end of some period of time $[0, T]$, the collected products are brought back to the first shop and will be stored as long as necessary and then repaired. If the repaired products are finished, the manufacturing process starts to cover the remaining demand for the time interval. The switching cost is incurred when the process shifts from repair to production and from production to repair. In the study of Saadany and Jaber (2008), the holding cost expression in Richter's model was modified because of the effect of the first time interval (see Fig. 1). This helps to reduce the total inventories of all the subsequent time intervals.

According to Saadany and Jaber (2008), the modified cost function in the model of Richter (1997) with switching costs is equal to

$$K_2(x,m,n,\alpha) = ((m-1)r + r_1 + (n-1)s + s_1) + \frac{h}{2d}\left(\frac{\alpha^2 x^2}{n} + \frac{\beta^2 x^2}{m}\right) + \frac{u\beta Tx}{2} - \frac{u\beta^2 x^2 (m-1)}{2dm}.$$

The modified cost per time unit function is obtained by dividing by $T$

$$K(x,m,n,\alpha) = \frac{K_2(x,m,n,\alpha)}{T} = \frac{d}{x}((m-1)r + r_1 + (n-1)s + s_1) + \frac{x}{2}\left[h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta - \frac{u\beta^2(m-1)}{m}\right], \quad (1)$$

where $x = dT$. The function (1) is convex and differentiable in $x$, therefore there is a unique minimum point

$$x(m,n,\alpha) = \sqrt{\frac{2d((m-1)r + r_1 + (n-1)s + s_1)}{h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta - u\beta^2\left(\frac{m-1}{m}\right)}}. \quad (2)$$

The minimum cost per time unit for given values $m, n, \alpha$ is obtained by substituting (2) into (1):

$$K(m,n,\alpha) = \sqrt{2d((m-1)r + r_1 + (n-1)s + s_1)\left(h\left(\frac{\alpha^2}{n} + \frac{\beta^2}{m}\right) + u\beta - \frac{u\beta^2(m-1)}{m}\right)}. \quad (3)$$

## 4. Determining the optimal policy for the generalized EOQ waste and disposal model

To determine the optimal policy means to find the optimal numbers $m$ and $n$ for the minimum cost found in the previous Section (3). In this section $\alpha$ will be a constant and not a variable. Therefore, the function (3) will be denoted just by $K(m,n)$. The problem of determining the optimal batch numbers takes the following form as a nonlinear integer optimization problem (4)

$$\min_{(m,n)} K(m,n), \quad m, n \in \{1, 2, \ldots\}. \quad (4)$$

The determination of optimal values for $m, n$ and later also $\alpha$, constitutes the problem of our paper and of other studies as well.

In order to derive explicit expressions for the optimal values in problem (4) let us first introduce the notations

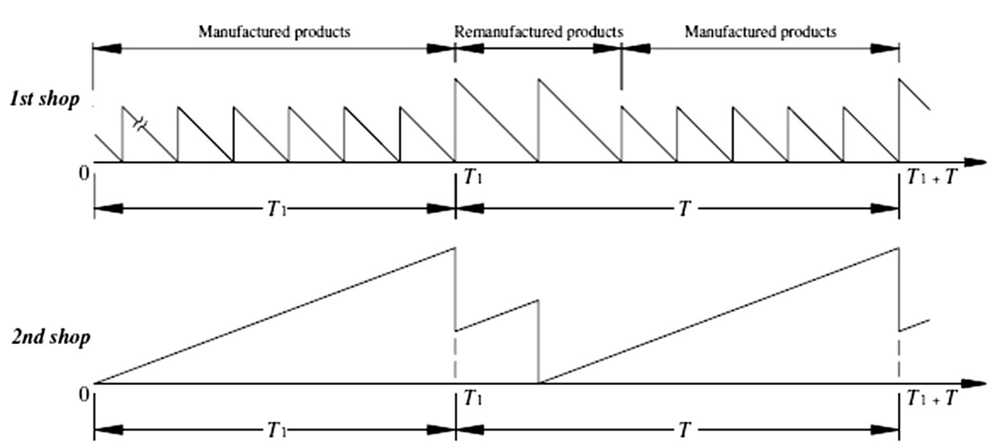


**Fig. 1.** The modified behavior of inventory in the 1st and 2nd shops.