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Generalized intuitionistic fuzzy interactive geometric interaction operators using Einstein t-norm and t-conorm and their application to decision making

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ABSTRACT

The present paper proposes some new geometric aggregation operations on the intuitionistic fuzzy sets (IFSs) environment. Based on it, a new class of generalized geometric interaction averaging aggregation operators using Einstein norms and conorms are developed, which includes the weighted, ordered weighted and hybrid weighted averaging operators. Furthermore, desirable properties corresponding to proposed operators have been stated. Finally, a multi-criteria decision making (MCDM) problem has been illustrated to show the validity and effectiveness of the proposed operators. The computed results have been compared with the existing results.

1. Introduction

Decision making is one of the most significant and omnipresent human activities in business, services, manufacturing, selection of products, etc. It is understandable that the different attributes in the decision problem are likely to play different roles in reaching a final decision. These varying roles are typically reflected at different attribute weights in MCDM. But due to the various constraints in day-today life, decision makers may give their judgements under the uncertain and imprecise in nature. Thus, there always exists a degree of hesitancy between the preferences of the decision making and hence, the analysis conducted under such circumstances is not ideal and hence does not tell the exact information to the system analyst. An IFS theory (Attanassov, 1986), which is an extension of the fuzzy set theory (Zadeh, 1965), is one of the most successful theory to handle and describe the uncertainties in the data in terms of defining their membership and non-membership grades corresponding to each element in the universe of discourse (Garg, 2013, 2016d; Garg, Rani, Sharma, & Vishwakarma, 2014; Xu, 2007a, 2007c). The information aggregation process is one of the important and interesting topic in the phase of the decision making and which is receiving more and more attention. The weighted average (WA) and ordered weighted average (OWA) (Yager, 1988; Yager & Kacprzyk, 1997), whose prominent characteristic is the reordering step, provides a parameterized family of aggregation operators. In that direction, geometric aggregation operators in the IFS environment has been presented by Xu and Yager (2006). Xu (2007a) proposed the various intuitionistic fuzzy aggregation operators. Zhao, Xu, Ni, and Liu (2010) combined Xu and Yager's operators and developed their corresponding generalized aggregation operators. Xia and Xu (2010) proposed a series of intuitionistic fuzzy point aggregation operators based on the generalized aggregation operators (Zhao et al., 2010). But it has been analyzed that the above operators used Archimedean t-norm and t-conorm for the aggregation process. An alter to this, Einstein based t-norm and t-conorm have a best approximation for sum and product the intuitionistic fuzzy numbers (IFNs). Wang and Liu (2012) proposed some geometric as well as averaging aggregation operator based on weighted and ordered weighted operators for different IFNs. Zhao and Wei (2013) extended their aggregation operators by using the hybrid average and geometric operators. Apart from this, various researchers pay more attention on IFSs for aggregating the different alternatives using different aggregation operators (Beliakov, Pradera, & Calvo, 2007; Deshrijver & Kerre, 2002; Garg, 2015, 2016a, 2016b, 2016c; Garg, Agarwal, & Tripathi, 2015; He et al., 2014; Liu, 2014; Wei, 2010; Xu & Da, 2003; Xu & Xia, 2003; Yu, 2015; Zhou & Chen, 2011; Zhou, Chen, Merigo, & Gil-Lafuente, 2012).

The above existing operational laws have been used by the various researchers but it has been observed that they are not capable to be used for most of the purposes. For example, consider two IFSs

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A and B such that either the grades of membership or non-membership of any set is zero, i.e. if either $\mu_A = 0$ and $\mu_B \neq 0$ or $\nu_A = 0$ and $\nu_B \neq 0$ then by multiplication law of intuitionistic fuzzy sets we have $\mu_{A\otimes B} = 0$ which states that μ_B does not play any significant role during the aggregation process. Similarly, by addition operations, we have $\nu_{A\oplus B} = 0$ which means that ν_B has no effect on the aggregation. Therefore, it can be concluded from the above study that there exists an undesirable feature of the existing operators. Another shortcoming of the current aggregation operators is that in their operations no dependency relation exists between the membership functions. Due to these features, their corresponding aggregation results are inconsistent to rank the alternatives. Hence, there is a need to modify these existing operations by considering the degree of membership functions properly.

Thus, the present work extends the existing approaches by taking into the account the pair of proper interactions between the membership functions by using the Einstein operation laws. Therefore, the objective is to present operational laws by defining the new operations on Einstein product and sum by taking the pair of μ_A, ν_B and ν_A, μ_B in the non-membership function. The major advantages of these pairs are that the grade of the non-membership is superior to the membership function, which shows the nature of decision maker to be pessimistic. Based on these laws, we propose some series of averaging geometric operators, namely, generalized intuitionistic fuzzy Einstein weighted geometric interaction averaging (GIFEWGIA), ordered weighted (GIFEOWGIA) and hybrid weighted (GIFEHWGIA). Finally, an MCDM method based on these proposed aggregation operators has been presented to show the applicability, utility and validity of the proposed ones. From the studies, it has been concluded that it can properly handle the shortcoming of the existing work and hence provides an alternative way to find the best alternative using aggregation operators.

The rest of the paper is organized as follows: Basic definitions related to the IFSs and existing aggregation operators on it are summarized in Section 2 along with the shortcomings of the existing operators. The operational laws of IFNs and the information aggregation methods based on these operational laws are discussed in Section 3. In Section 4, a method for solving MCDM problem has been presented using proposed aggregation operators where attributed values are represented in the forms of IFNs. A numerical example is illustrated in Section 5 to show the superiority and validity of the new approach. Finally, Section 6 concludes the paper.

2. Preliminaries

2.1. Intuitionistic fuzzy set

An intuitionistic fuzzy set (IFS) A in a finite universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ is (Attanassov, 1986)

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$$

where $\mu_A, \nu_A : X \to [0,1]$ represents, respectively, the grades of membership and non-membership corresponding to the element x with the conditions $0 \leqslant \mu_A(x), \nu_A(x) \leqslant 1$, and $\mu_A(x) + \nu_A(x) \leqslant 1$. For convenience, the pair $(\mu_A(x), \nu_A(x))$ is called an intuitionistic fuzzy number (IFN) (Xu & Yager, 2006; Xu, 2007a). A score S and an accuracy function H (Xu, 2007a) of IFN A can be represented as $S(A) = \mu_A - \nu_A$ and $H(A) = \mu_A + \nu_A$. Based on these functions, an ordered relation between two IFNs, $A_1 = \langle \mu_1, \nu_1\rangle$ and $A_2 = \langle \mu_2, \nu_2\rangle$, is defined as follows (Xu & Yager, 2006; Xu, 2007a).

(i) If $S(A_1) > S(A_2)$ then $A_1 \succ A_2$.
(ii) If $S(A_1) < S(A_2)$ then $A_1 \prec A_2$.
(iii) If $S(A_1) = S(A_2)$ then
- If $H(A_1) > H(A_2)$ then $A_1 \succ A_2$;
- If $H(A_1) = H(A_2)$ then $A_1 = A_2$.

As for the IFNs, Xu (2007a) defined the following operators for aggregating the family of IFNs $(\alpha_1, \alpha_2, \ldots, \alpha_n)$ by using weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of α_i $(i = 1, 2, \ldots, n)$ and $\omega_i > 0$ and $\sum_{i=1}^n \omega_i = 1$.

(a) Intuitionistic fuzzy weighted geometric (IFWG) operator

$$IFWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha_1^{\omega_1} \otimes \alpha_2^{\omega_2} \otimes \cdots \otimes \alpha_n^{\omega_n} = \left\langle \prod_{i=1}^n (\mu_i)^{\omega_i}, 1 - \prod_{i=1}^n (1-\nu_i)^{\omega_i} \right\rangle$$

(b) Intuitionistic fuzzy ordered weighted geometric (IFOWG) operator

$$IFOWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \alpha_{\sigma(1)}^{\omega_1} \otimes \alpha_{\sigma(2)}^{\omega_2} \otimes \cdots \otimes \alpha_{\sigma(n)}^{\omega_n} = \left\langle \prod_{i=1}^n (\mu_{\sigma(i)})^{\omega_i}, 1 - \prod_{i=1}^n (1-\nu_{\sigma(i)})^{\omega_i} \right\rangle$$

where $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $\alpha_{\sigma(i-1)} \geqslant \alpha_{\sigma(i)}$ for all $i = 1, 2, \ldots, n$

(c) Intuitionistic fuzzy hybrid geometric (IFHG) operator

$$IFHG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \dot{\alpha}_{\delta(1)}^{\omega_1} \otimes \dot{\alpha}_{\delta(2)}^{\omega_2} \otimes \cdots \otimes \dot{\alpha}_{\delta(n)}^{\omega_n} = \left\langle \prod_{i=1}^n (\dot{\mu}_{\delta(i)})^{\omega_i}, 1 - \prod_{i=1}^n (1-\dot{\nu}_{\delta(i)})^{\omega_i} \right\rangle$$

where $\dot{\alpha}_{\delta(i)}$ is the ith largest of the weighted intuitionistic fuzzy values $\dot{\alpha}_i$ $(\dot{\alpha}_i = \alpha_i^{nw_i}, i = 1, 2, \ldots, n)$

Based on the definition of IFWG operator, Tan, Yi, and Chen (2015) introduced the generalized intuitionistic fuzzy weighted geometric (GIFWG) aggregation operators and is defined as follows.

$$GIFWG(\alpha_1, \alpha_2, \ldots, \alpha_n) = \frac{1}{\lambda}\left((\lambda\alpha_1)^{\omega_1} \otimes (\lambda\alpha_2)^{\omega_2} \otimes \cdots \otimes (\lambda\alpha_n)^{\omega_n}\right) = \left\langle 1 - \left(1 - \prod_{i=1}^n \left(1 - (1-\mu_i)^{\lambda}\right)^{\omega_i}\right)^{1/\lambda}, \left(1 - \prod_{i=1}^n (1-\nu_i^{\lambda})^{\omega_i}\right)^{1/\lambda} \right\rangle$$

where $\lambda > 0$ is a real number. Furthermore, if $\lambda = 1$, then IFWG operator is a special case of GIFWG operator.

2.2. Einstein operations of IFS

Einstein operations are one of the t-norm and t-conorms which includes the Einstein sum and product, respectively, as

$$a \oplus b = \frac{a+b}{1+ab}; \quad a \otimes b = \frac{a \cdot b}{1 + (1-a)\cdot(1-b)}, \quad \forall\ (a,b) \in [0,1]^2$$

Based on these, the algebraic operations defined on three IFNs $\alpha = (\mu, \nu), \alpha_1 = (\mu_1, \nu_1)$ and $\alpha_2 = (\mu_2, \nu_2)$ and $\lambda > 0$ be a real number, as (Wang & Liu, 2012)

(i) $\alpha_1 \otimes \alpha_2 = \left\langle \frac{\mu_1\mu_2}{1+(1-\mu_1)(1-\mu_2)}, \frac{\nu_1+\nu_2}{1+\nu_1\nu_2} \right\rangle$
(ii) $\alpha_1 \oplus \alpha_2 = \left\langle \frac{\mu_1+\mu_2}{1+\mu_1\mu_2}, \frac{\nu_1\nu_2}{1+(1-\nu_1)(1-\nu_2)} \right\rangle$
(iii) $\lambda\alpha = \left\langle \frac{(1+\mu)^{\lambda}-(1-\mu)^{\lambda}}{(1+\mu)^{\lambda}+(1-\mu)^{\lambda}}, \frac{2\nu^{\lambda}}{(2-\nu)^{\lambda}+\nu^{\lambda}} \right\rangle$
(iv) $\alpha^{\lambda} = \left\langle \frac{2\mu^{\lambda}}{(2-\mu)^{\lambda}+\mu^{\lambda}}, \frac{(1+\nu)^{\lambda}-(1-\nu)^{\lambda}}{(1+\nu)^{\lambda}+(1-\nu)^{\lambda}} \right\rangle$