



Multiple attribute group decision making based on interval-valued hesitant fuzzy information measures



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ABSTRACT

Under the interval-valued hesitant fuzzy environment, we investigate a multiple attribute group decision making (MAGDM) method on the basis of some information measures. We first introduce three axiomatic definitions of information measures under interval-valued hesitant fuzzy environment, including the entropy, similarity measures and cross-entropy. Several information measure formulas for interval-valued hesitant fuzzy elements (IVHFEs) are further constructed, which is based on the continuous ordered weighted averaging (COWA) operator. Then, the relationship among the entropy, similarity measures and cross-entropy is discussed, from which we find that three information measures can be transformed by each other based on their axiomatic definitions. The programming model is established to determine optimal weight of attribute with the principle of minimum entropy and maximum cross-entropy. Furthermore, an approach to MAGDM is developed, in which the attribute values take the form of IVHFEs. Finally, a numerical example for emergency risk management (ERM) evaluation is provided to illustrate the application of the developed approach.

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1. Introduction

Since Zadeh introduced the fuzzy sets (FSs) (Zadeh, 1965), it has achieved a great success in various fields. The concept of intuitionistic fuzzy sets (IFSs) (Atanassov, 1986, 1989, 2000) put forward by Atanassov, which is a generalization of the FSs. Atanassov and Gargov further introduced the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) (Atanassov & Gargov, 1989), whose components are intervals rather than exact numbers. The introduction of IFSs and IVIFSs proved to be very meaningful and practical, and have been found to be highly useful to cope with uncertainty and vagueness (Yager, 2009; Xu, 2007; Zhou, Tao, Chen, & Liu, 2014; Chen & Li, 2010; Xia & Xu, 2010; Li, 2010; Zhou, Jin, Chen, & Liu, 2016). However, in the process of decision making, decision makers (DMs) are usually irresolute and hesitant for one thing or another, which makes it difficult to determine the membership of an element to a set due to doubts between a few different values. In this case, Torra and Narukawa (2009) and Torra (2010) proposed the hesitant fuzzy set (HFS) considered as another generalization of FSs, which permits the membership having a collection of possible

values. To accommodate more complex environment, several extensions of the HFSs have been developed, such as interval-valued hesitant fuzzy sets (IVHFSs) (Chen, Xu, & Xia, 2013a, 2013b), dual hesitant fuzzy sets (DHFSs) (Zhu, Xu, & Xia, 2012), generalized hesitant fuzzy sets (GHFSs) (Qian, Wang, & Feng, 2013) and interval type-2 hesitant fuzzy sets (IT2HFSs) (Hu, Xiao, Chen, & Liu, 2015). Especially, considering that the interval-valued fuzzy set (IVFS) (Turksen, 1986) is usually more adequate or sufficient to real-life decision making problems than real numbers, Chen et al. (2013a), (2013b) introduced the concept of IVHFS, which permits the membership having a collection of possible interval-valued numbers.

Entropy, similarity measures, and cross-entropy are three important research topics in the fuzzy theory, which have been widely used in practical applications (Xu & Xia, 2012; Zadeh, 1968; De Luca & Termini, 1972; Szmidt & Kacprzyk, 2001; Ye, 2010; Wei, Wang, & Zhang, 2011; Grzegorzewski, 2004; Hung & Yang, 2004, 2007; Zhou, Tao, Chen, & Liu, 2014; Mitchell, 2003; Xu & Chen, 2008; Xu & Yager, 2009; Liu, 1992; Vlachos & Sergiadis, 2007; Xia & Xu, 2012), such as decision-making, clustering analysis, medical diagnosis, and pattern recognition. Entropy is the measure of fuzziness. Since its appearance, entropy has received great attentions. The fuzzy entropy was first introduced by Zadeh (1968). Moreover,

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De Luca and Termini (1972) presented the axioms with which the fuzzy entropy should comply, and defined the entropy of a FS. Based on the ratio of intuitionistic fuzzy cardinalities, Szmidt and Kacprzyk (2001) given the axiomatic requirements of intuitionistic fuzzy entropy measure and introduced a non-probabilistic-type entropy measure for IFs. Ye (2010) proposed two entropy measures for IVIFSs and established an entropy weighted model to determine the entropy weights with respect to a decision matrix provided as IVIFSs. Wei et al. (2011) developed an entropy measure for IVIFSs, which generalized three entropy measures for IFs. Xu and Xia (2012) introduced the concepts of entropy and cross-entropy for HFSs, and discussed their desirable properties.

Similarity measures and cross-entropy are mainly used to measure the discrimination information. Up to now, a lot of research has been done about this issue (Grzegorzewski, 2004; Hung & Yang, 2004, 2007; Zhou et al., 2014; Mitchell, 2003; Xu & Chen, 2008; Xu & Yager, 2009). Liu (1992) gave the axiomatic definitions of entropy, distance measure, and similarity measure of FSs and systematically discussed their basic relations. Vlachos and Sergiadiis (2007) introduced the concept intuitionistic fuzzy cross-entropy, and discussed relations between cross-entropy and entropy. Based on entropy and cross-entropy, Xia and Xu (2012) developed two pairs of entropy and cross entropy measures for IFs and applied them to MAGDM. Beliakov, Pagola, and Wilkin (2014) investigated a new approach for defining similarity measures for IFs, in which a similarity measure has two components indicating the similarity and hesitancy aspects. Based on the COWA operator (Yager, 2004), Jin, Pei, Chen, and Zhou (2014) proposed an interval-valued intuitionistic fuzzy continuous weighted entropy, and then developed an approach to deal with interval-valued intuitionistic fuzzy MAGDM problems. The relationships among the entropy, similarity measures and cross-entropy have also attracted many attentions. Zhang, Zhang, and Mei (2009) and Zeng & Li (2006) showed that entropies and similarity measures of IVFSs can be transformed by each other. Farhadinia (2014) studied the relationship between the entropy, similarity measure and distance measure for HFSs and IVHFSs, and developed two clustering algorithms under hesitant fuzzy environment. Hu, Zhang, Chen, and Liu (2016) presented more reasonable information measures for HFSs, and then proposed a new TOPSIS method under the hesitant fuzzy environment. Xu and Xia (2012) analyzed the relationships among entropy, cross-entropy, and similarity measures under hesitant fuzzy environment.

From above analysis, we can see that information measures are very useful tools to cope with uncertainty and vagueness. On the one hand, it is known that hesitancy and complexity are the very common problems in human decision making process. Therefore, just as FSs, IFs, IVIFSs and HFSs, researches on the entropy, similarity measures and cross-entropy for IVHFSs are the important issues. On the other hand, more and more MAGDM methods and theories have been developed on the basis of IVHFSs. To the best of our knowledge, the aforementioned information measures, however, cannot be used to deal with the entropy, similarity measures and cross-entropy for IVHFSs and there are few studies focused on the relationship between the entropy, similarity measures and cross-entropy for IVHFSs. Therefore, it is necessary and meaningful to study some issues. For example, what is it like the expression of the interval-valued hesitant fuzzy information measures? What is the relationship among the interval-valued hesitant fuzzy information measures?

Motivated by the concepts of hesitant fuzzy entropy, similarity measures and cross-entropy (Xu & Xia, 2012), we introduce three axiomatic definitions of information measures for IVHFEs, and then we propose some information measure formulas based on the COWA operator. The relationship among these information measures for IVHFEs is discussed. Moreover, we investigate an approach to MAGDM based on the proposed information measures for IVHFEs.

To do this, the rest of the paper is organized as follows. In Section 2, we briefly review the concepts of HFSs and IVHFSs. Section 3 introduces the axiomatic definitions of entropy, similarity measures and cross-entropy for IVHFEs, and constructs the interval-valued hesitant fuzzy continuous entropy formulas, continuous similarity measure formulas and continuous cross-entropy formulas based on the COWA operator. In Section 4, we prove that the entropy, the similarity measure and the cross-entropy of the IVHFE can be transformed by each other based on their axiomatic definitions. Section 5 develops an approach to MAGDM based on the continuous entropy, continuous similarity and continuous cross-entropy. In Section 6, a numerical example of emergency operating center evaluation is provided to illustrate the application of the developed method. Finally, we end the paper by summarizing the main conclusions in Section 7.

2. Preliminaries

2.1. HFSs and the entropy of HFEs

In the following, we reviews some basic concepts related to HFSs and the axiomatic definition of entropy for hesitant fuzzy elements (HFEs) (Xia & Xu, 2011).

Definition 2.1 (Torra, 2010). Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed set, a HFS A on X is defined in terms of a function $\alpha_A(x_i)$ that returns a subset of $[0, 1]$ when it is applied to x_i , i.e.,

$$A = \{\langle x_i, \alpha_A(x_i) \rangle | x_i \in X\} \quad (1)$$

where $\alpha_A(x_i)$ is a set of some different values in $[0, 1]$, representing the possible membership degrees of the element $x_i \in X$ to the set A . For convenience, we refer to $\alpha = \alpha_A(x_i)$ as an HFE, which is a basic unit of HFS. Let H be the set of all the HFEs on X .

For an HFE α , Xu and Xia first axiomatized hesitant fuzzy entropy measure.

Definition 2.2 (Xu & Xia, 2012). An entropy on HFE α is a real-valued function $I : H \rightarrow [0, 1]$, which satisfying the following axiomatic requirements:

- (1) $I(\alpha) = 0$, if and only if $\alpha = 0$ or $\alpha = 1$;
- (2) $I(\alpha) = 1$, if and only if $\alpha_{(j)} + \alpha_{(l_x-j+1)} = 1$, for $j = 1, 2, \dots, l$;
- (3) $I(\alpha) = I(\alpha^c)$, where $\alpha^c = \{1 - \gamma | \gamma \in \alpha\}$;
- (4) $I(\alpha) \leq I(\beta)$, if $\alpha_{(j)} \leq \beta_{(j)}$, for $\beta_{(j)} + \beta_{(l_x-j+1)} \leq 1$ or $\alpha_{(j)} \geq \beta_{(j)}$ for $\beta_{(j)} + \beta_{(l_x-j+1)} \geq 1$, for $j = 1, 2, \dots, l$.

2.2. IVHFSs and COWA operator

In fact, it may be difficult for decision makers (DMs) to quantify their preference value with a crisp number in many MAGDM problems, but they can be represented by an interval number within $[0, 1]$. Therefore, Chen et al. (2013a), (2013b) introduced the concept of IVHFSs that will be required for our subsequent developments.

Definition 2.3. Let $X = \{x_1, x_2, \dots, x_n\}$ be a finite set, and $D[0, 1]$ be the set of all closed subintervals of $[0, 1]$. An IVHFS on X is

$$\tilde{A} = \{\langle x_i, \tilde{\alpha}_A(x_i) \rangle | x_i \in X\} \quad (2)$$

where $\tilde{\alpha}_A(x_i) : X \rightarrow D[0, 1]$ is a set of all possible interval-valued membership degrees of the element $x_i \in X$ to the set $\tilde{A}$. We refer to $\tilde{\alpha}_i = \tilde{\alpha}_A(x_i) = \{[\tilde{\gamma}_i^-, \tilde{\gamma}_i^+] | \tilde{\gamma}_i \in \tilde{\alpha}_A(x_i)\}$ as an IVHFE, where $\tilde{\gamma}_i^- = \inf \tilde{\gamma}_i$

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