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Initial value problem formulation of time domain boundary element method for electromagnetic microwave simulations

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ABSTRACT

To aim to obtain more stable solutions and wider area applications for the Time Domain Boundary Element Method (TDBEM), initial value problem formulation of the TDBEM is newly introduced for microwave simulations. The initial value problem formulation of the TDBEM allows us to solve transient microwave phenomena as interior region problems, which gives us well matrix property and interior resonance free solutions. This paper concentrates on applying the initial value problem formulation of the TDBEM to wake field phenomena in particle accelerator cavities.

1. Introduction

In general, the Boundary Element Method (BEM) is a time and memory consuming scheme compared with other numerical methods for Partial Differential Equations (PDE) such as the Finite Difference Method (FDM) and the Finite Element Method (FEM). Especially the Time Domain Boundary Element Method (TDBEM) is much more time and memory consuming scheme than the standard BEM due to iterative matrix calculations in time domain. Therefore we should carefully select target applications for the BEM, which bring us advantages of the BEM, for example, open boundary problems, moving boundary problems, inverse problems, shape optimization, coupling problems with moving charged particles. Authors have been working in development of the TDBEM for analysis of the so-called "wake fields", which are the electromagnetic microwave produced by moving charged particles in particle accelerator cavity [1–15]. Then the Finite Difference Time Domain (FDTD) method or the Finite Integration Technique (FIT) is commonly used for the wake field analysis [16] and can simulate the microwave phenomena much faster with smaller memory than the TDBEM. Although the TDBEM has such disadvantages in memory requirement and calculation time, the TDBEM provides us attractive advantages of dispersion free property and treatment of curved particle trajectory in particle accelerator wake field analysis due to its original feature of surface meshing. In particle accelerator science, a proper method of the so-called "moving window scheme" is well-known for memory and CPU time reduction [17], since they are interested in only influence of the wake field on the charged particle dynamics. On the other hand, it is assumed that the boundary surfaces are all perfectly electric conductor (PEC) and initial values of the time domain calculation are all zero [18–22] in the conventional EFIE (Electric Field Integral Equation) and MFIE (Magnetic Field Integral Equation), which are fundamental equations for both formulations of frequency and time domain BEM. Then, initial value of electromagnetic fields is needed in the moving window scheme and therefore it is not easy to introduce the moving window scheme into the TDBEM.

This paper presents a more general formulation, initial value problem formulation of the TDBEM for microwave simulations. In the initial value problem formulation of the TDBEM, the assumptions of the PEC boundary condition and the zero initial value are removed and the TDBEM simulations can be applied for any kind of four dimensional domains. To use the initial value problem formulation, the moving window scheme can be smoothly combined with the TDBEM and enable us to do wake field analysis for long structure accelerator cavities.

2. Formulation

The initial value problem formulation of the TDBEM is derived by means of a standard procedure of the BEM formulation, and

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then the conventional TDBEM can be understood as simplification of the initial value problem formulation of the TDBEM. To distinguish differences from the conventional TDBEM, we shortly summarize the conventional formulation, and then proceed to the initial value problem formulation.

2.1. *Conventional formulation of time domain boundary element method [9–12]*

We start the formulation from the following Green's theorem in four dimensional space-time for any two independent (scalar) functions Φ, and Ψ as follows [23]:

$$\int_\Omega \left(\frac{\partial^2 \Phi}{\partial x_\lambda \partial x^\lambda} \Psi - \Phi \frac{\partial^2 \Psi}{\partial x_\lambda \partial x^\lambda} \right) d\Omega = \int_{S+V_1+V_0} \left(\frac{\partial \Phi}{\partial x^\nu} \Psi - \Phi \frac{\partial \Psi}{\partial x^\nu} \right) dV^\nu \tag{1}$$

where Ω is the four dimensional region, x_λ and x^λ are covariant and contravariant 4D coordinate vectors. We are assuming that the boundary shape does not change during time step simulation, so the 4D region is tube structure, which is parallel to the time axis (see Fig. 1). Then V_0 and V_1 denote the top and bottom surface of Ω (cross section subspace of Ω at $t=t_0$ and t_1), and S is the side surface of Ω. And then $d\Omega = cdtdxdydz = cdtdV$ is an infinitesimal four dimensional volume, dV is pure three dimensional volume, and dV^ν is outer normal infinitesimal surface element 4D vector on the closed surface of the four dimensional region Ω:

$$\begin{aligned} dV^\nu &= (dxdydz, cdtdydz, cdtdzdx, cdtdxdy) \\ &\equiv (dV, cdtd\mathbf{S}) \end{aligned} \tag{2}$$

where c is the velocity of light, $d\mathbf{S}$ is the outer normal infinitesimal surface element vector on the closed surface of the three dimensional region V. Substituting a solution $A(ct,\mathbf{x})$ of the following inhomogeneous wave equation:

$$\left(\nabla^2 - \frac{\partial^2}{c^2 \partial t^2} \right) A(ct,\mathbf{x}) = -\mu J(ct,\mathbf{x}) \quad \text{or} \quad \frac{\partial^2 A}{\partial x_\lambda \partial x^\lambda} = \mu J, \tag{3}$$

and the following fundamental solution $G(ct,\mathbf{x})$ of the inhomogeneous wave equation to Φ and Ψ in (1):

$$G(ct,\mathbf{x};ct',\mathbf{x}') = \frac{1}{4\pi} \frac{\delta(t'-t+(|\mathbf{x}-\mathbf{x}'|/c))}{|\mathbf{x}-\mathbf{x}'|}, \tag{4}$$

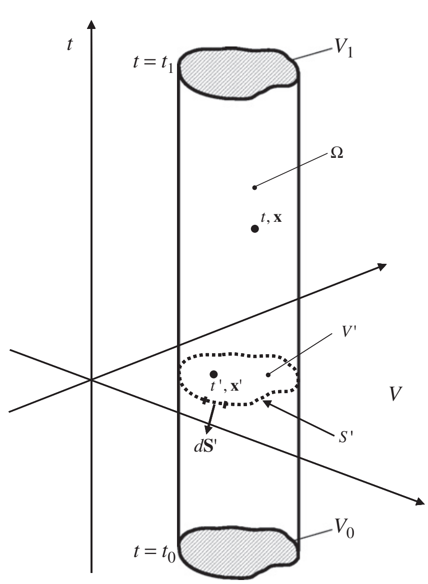

Fig. 1. Time invariant domain in 4D space time.

we obtain the following integral equation:

$$\begin{aligned} A(ct,\mathbf{x}) = {} & \frac{\mu}{4\pi} \int_V \frac{J\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|} dV' \\ & + \frac{1}{4\pi} \int_S \left(\frac{1}{|\mathbf{x}-\mathbf{x}'|} \frac{\partial A\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{\partial n} \right. \\ & \left. - \left(\frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^2} \frac{\partial}{c \partial t'} \right) A(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}') \right) dS' \\ & - \frac{1}{4\pi} \int_{V_1+V_0} \left(\frac{\partial A(ct',\mathbf{x}')}{c \partial t'} \frac{\delta\left(t'-t+\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{|\mathbf{x}-\mathbf{x}'|} \right. \\ & \left. - A(ct',\mathbf{x}') \frac{\dot{\delta}\left(t'-t+\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{c|\mathbf{x}-\mathbf{x}'|} \right) dV' \end{aligned} \tag{5}$$

where $\mathbf{n}$ is outer unit normal vector on boundary surface, μ is permeability and $\dot{\delta}$ denotes the ordinary derivative $d\delta(t)/dt$. Then the volume integral on V_1 in the third term of the right hand side vanishes for $t' > t$ due to the causality property of the delta function:

$$\begin{aligned} A(ct,\mathbf{x}) = {} & \frac{\mu}{4\pi} \int_V \frac{J\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|} dV' \\ & + \frac{1}{4\pi} \int_S \left(\frac{1}{|\mathbf{x}-\mathbf{x}'|} \frac{\partial A\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{\partial n} \right. \\ & \left. - \left(\frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^2} \frac{\partial}{c \partial t'} \right) A\left(t - \frac{|\mathbf{x}-\mathbf{x}'|'}{c}, \mathbf{x}'\right) \right) dS' \\ & + \frac{1}{4\pi} \int_{V_0} \left(\frac{\partial A(ct',\mathbf{x}')}{c \partial t'} \frac{\delta\left(t'-t+\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{|\mathbf{x}-\mathbf{x}'|} \right. \\ & \left. - A(ct',\mathbf{x}') \frac{\dot{\delta}\left(t'-t+\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)}{c|\mathbf{x}-\mathbf{x}'|} \right) dV' \end{aligned} \tag{5'}$$

The third term of the right hand side of (5′) expresses contribution of the initial value of electromagnetic fields in 3D domain volume. In the third term, all functions in the integrand are evaluated at the initial time $t' = t_0$. If we assume that any field value at the initial time $t = t_0$ are all zero, (5′) reduces to

$$\begin{aligned} A(ct,\mathbf{x}) = {} & \frac{\mu}{4\pi} \int_V \frac{J\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|} dV' \\ & + \frac{1}{4\pi} \int_S \left(\frac{1}{|\mathbf{x}-\mathbf{x}'|} \frac{\partial A\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{\partial n} \right. \\ & \left. - \left(\frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^2} \frac{\partial}{c \partial t'} \right) A\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right) \right) dS' \end{aligned} \tag{6}$$

Application of this integral equation to scalar potential φ and vector potential $\mathbf{A}$ yields

$$\begin{aligned} \varphi(ct,\mathbf{x}) = {} & \frac{1}{4\pi\varepsilon} \int_V \frac{\rho\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|} dV' \\ & + \frac{1}{4\pi} \int_S \left(\frac{1}{|\mathbf{x}-\mathbf{x}'|} \frac{\partial \varphi\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{\partial n} \right. \\ & \left. - \left(\frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^3} + \frac{(\mathbf{x}-\mathbf{x}') \cdot \mathbf{n}}{|\mathbf{x}-\mathbf{x}'|^2} \frac{\partial}{c \partial t'} \right) \varphi(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}') \right) dS' \end{aligned} \tag{7}$$

$$\mathbf{A}(ct,\mathbf{x}) = \frac{\mu}{4\pi} \int_V \frac{\mathbf{J}\left(t - \frac{|\mathbf{x}-\mathbf{x}'|}{c}, \mathbf{x}'\right)}{|\mathbf{x}-\mathbf{x}'|} dV'$$