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Dual frontiers without convexity

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ABSTRACT

The conventional optimistic data envelopment analysis (DEA) model typically evaluates decision-making units (DMUs) using the best relative efficiency that is derived from the estimated efficient production frontier, while the pessimistic DEA model evaluates the DMUs according to the estimated inefficient production frontier. Although the results of these models vary significantly, both approaches should be incorporated for an overall performance evaluation, and a significant body of literature exists on this topic. This paper contributes to the literature by providing entirely new non-convex optimistic and pessimistic models by applying free disposal hull (FDH) technology, which is important in real-life scenarios. These models may experience a lack of sufficient discrimination power. Accordingly, two improved versions of both approaches are developed. The first version formulates the models in the presence of the slack variables. In the second version, we propose FDH super-efficiency models that may become infeasible. Thus, we propose the modified models without infeasibility problem. The paper concludes with a comprehensive empirical study to illustrate the details and applicability of the proposed models.

1. Introduction

The aim of frontier analysis is to construct the empirical analogue of the production function by the use of the production possibility set (PPS)-also so-called *technology*-whose frontier is used to evaluate the firms (Grosskopf, 1986). Data envelopment analysis (DEA), a well-known non-parametric method in frontier analysis for measuring the relative efficiency between decision-making units (DMUs), was introduced more than 35 years ago when Charnes, Cooper, and Rhodes (1978) presented their so-called *CCR* model for constant returns to scale (CRS), through which they were able to construct the PPS based on the mathematical programming techniques. The main idea behind the original DEA models is that an empirical best practice frontier is first constructed by enveloping the observed data through a minimal spanning hull, and then the efficiency measure is determined based on radial projection to the production frontier. DEA has since attracted the attention of many researchers because of its unique ability to measure the efficiency of multiple-input and multiple-output DMUs without assigning prior weights to the inputs and outputs, resulting in the proposal of a wide range of DEA models (Cook & Seiford, 2009). Indeed, empirical applications of DEA can be found in many sectors, including education (Bessent, Bessent, Kennington, & Reagan, 1982), banking (Emrouznejad, Rostamy-Malkhalifeh, Hatami-Marbini, & Tavana, 2012; Thanassoulis, 1999), manufacturing (Wahab, Wu, & Lee, 2008), logistics (Xu, Li, & Wu, 2009), telecommunication (Cooper, Park, & Yu, 2001), healthcare (Jacobs, 2001), and even sports (Cooper, Ruiz, & Sirvent, 2009; Sexton & Lewis, 2003).

A remarkable DEA model developed in the literature is the free disposal hull (FDH) that is based on exceptionally powerful line of reasoning. Deprins, Simar, and Tulkens (1984) were the first to propose FDH model with a non-convex technology, and FDH was further developed by Tulkens (1993). FDH is different from the DEA family in that it requires the minimal satisfaction on the assumptions for creating the 'staircase' shape of the FDH frontier production. That is, it does not require convexity and/or proportionality assumptions. Although fewer studies have been conducted on the FDH model than on classical DEA, FDH is considered a more justifiable orientation from the practical and theoretical views than the hypothesized convex assumption in DEA (Tulkens & Eeckaut, 2006; Van Puyenbroeck, 1998).

Another specific characteristic of DEA models, so-called *optimism*, is to seek the most desirable input and output weights of a particular DMU with the aim of radially projecting on the efficiency production frontier, making each unit appear in its most favorable

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light. Considerable research has been conducted regarding this characteristic of DEA. However, many DEA researchers argued that using a pessimistic perspective in addition to the optimistic approach is important to render an equitable evaluation. The pessimistic DEA simply evaluates DMUs by constructing the inefficiency production frontier based on the least desirable weights to achieve the full inefficiency scores. A current body of literature exists regarding the incorporation of the pessimistic and optimistic approaches to achieve an unbiased evaluation (a literature review is provided in the following section of the paper).

According to the DEA literature on the double optimistic and pessimistic frontiers, none of the existing models accommodate non-convex frontiers. As discussed by Cherchye, Kuosmanen, and Post (2001) and Agrell and Tind (2001), non-convex frontiers are considered as an important technology that may be closer to the real-life situation, where we can relax the convexity assumption and no hypothetical frontiers need to be constructed. Moreover, conventional DEA leads to the indivisibility of input and output, and in many cases, the convexity axiom may be broken. More discussion and empirical evidence can be found in the work of Farrell (1959), Deprins et al. (1984), Tulkens (1993) and Kuosmanen (2001). In this paper, we propose an interval DEA model with double frontiers without convexity assumption by constructing optimistic and pessimistic FDH models to overcome the shortfall of optimism and the convexity of conventional DEA. After proposing the fundamental optimistic and pessimistic FDH models, we noticed that these models lack strong discrimination power among DMUs. Thus, the slack-based FDH models and super-efficiency FDH models were developed from both optimistic and pessimistic perspectives to overcome the discrimination power issue.

The remainder of this paper is organized as follows. The next section presents a literature review of the major related research studies that incorporate optimistic and pessimistic approaches. In Section 3, the axiomatic foundation of the optimistic DEA model is presented to develop the estimate formulation of the directional distance function from the input and output-orientations. Section 4 discusses the pessimistic directional distance function for both the input- and output-oriented models. In Section 5, we develop the optimistic and pessimistic FDH models for input- and output-oriented cases as well as extending the models to include the slack-based models and super-efficiency models to rank the efficient units. Section 6 modifies the super-efficiency FDH models to deal with the infeasibility problem followed by an empirical study in Section 7 to demonstrate the applicability of the proposed models. The empirical study shows the applicability and the results of our proposed models and compares them to the results to other models in the literature. In Section 8, our study is summarized with concluding remarks.

2. Literature review

There has been a series of research studies focusing on the development of classical DEA models while incorporating the pessimistic approach to achieve a better evaluation of the DMU under assessment. The main finding from this series of papers is that using only optimistic or only pessimistic DEA models is biased because there are efficient DMUs under the optimistic DEA, whereas they are inefficient under the pessimistic DEA, as shown in Fig. 1. Yamada, Matsui, and Sugiyama (1994) were the first authors to propose a pessimistic method to evaluate DMUs, which they termed inverted DEA (IDEA). All research work that incorporates optimistic DEA with pessimistic DEA can be categorized into two categories. The first category is called *interval efficiency*, where the efficiency score of each DMU in the dataset is calculated as an interval between the optimistic and pessimistic frontiers. There are

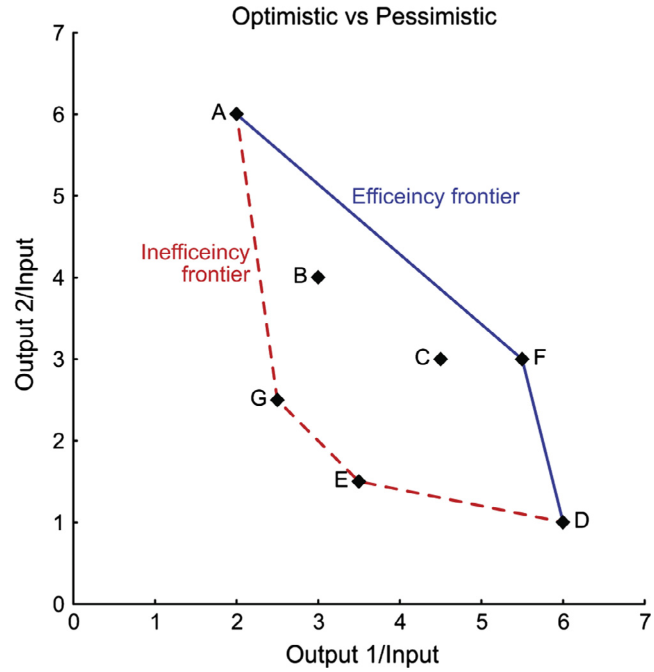

Fig. 1. Efficiency and inefficiency frontiers.

more studies devoted to this category than to the other one. The second category includes different approaches that assimilate both optimistic and pessimistic measures by directly applying some mathematical or statistical methods, such as averaging, virtual DMUs, or cross efficiency, with the aim of combining both measures.

The following paragraphs review the articles and research work related to the first category, namely, interval efficiency between optimistic and pessimistic DEA.

Doyle, Green, and Cook (1995) and Entani, Maeda, and Tanaka (2002) were among the first to combine the inverted or pessimistic DEA with the conventional optimistic DEA. Their research resulted in an efficiency score that is obtained based on an efficiency interval in which the lower bound is the pessimistic score and the upper bound is the optimistic score. Entani et al. (2002) constructed an interval model to calculate the efficiency, and these intervals are obtained from both optimistic and pessimistic scores. The model of Entani et al. (2002). can be considered the foundation model for the efficiency interval approach, where the final efficiency score for any DMU is denoted as an interval of the lower and upper limit efficiencies. Their model was initially proposed for crisp data and was extended to consider interval data and fuzzy data. A major shortfall is that it measures the pessimistic efficiency of each DMU under evaluation using only one input and one output. Entani and Tanaka (2006) tried to improve upon this model by adjusting the input and output of the data to make the upper bound of the efficiency interval equal 1 and the lower bound as large as possible to achieve the optimal evaluation. To overcome the shortfall of Entani et al. (2002), Wang and Yang (2007) introduced a method founded on the concept of virtual DMU called *anti-ideal DMU* (ADMU), which can be defined as a DMU that uses the maximum input value to produce the minimum output value. In their model, the efficiencies of all DMUs are obtained by calculating the worst and best performances of each DMU, and the interval efficiencies are calculated. Then, they used the Hurwicz criterion approach to rank each DMU based on these interval values. This model is referred to as a bounded DEA model because the efficiency of the DMU in question is bounded between upper