Computers & Industrial Engineering 101 (2016) 490–503

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

An improved Lagrangian relaxation heuristic for the scheduling problem of operating theatres

Zhou Bing-hai *, Yin Meng, Lu Zhi-qiang

School of Mechanical Engineering, Tongji University, Shanghai 201804, PR China

A R T I C L E   I N F O

Article history:
Received 15 October 2014
Received in revised form 28 January 2016
Accepted 1 September 2016
Available online 3 September 2016

Keywords:
Operating theatre scheduling
Bi-objective
Lagrangian relaxation algorithm
Branch and bound

A B S T R A C T

Due to the greatest importance of operating theatres in hospitals, this paper focuses on generating an optimal surgery schedule of elective-patients in multiple operating theatres, which considers three stages: the preoperative, perioperative, and postoperative stage. The scheduling problems of operating theatres allocate hospital resources to individual surgical cases and decide on the time to perform the surgeries in each stage. The problem consists of assigning patients to different resources in any surgical stage in order to minimize related costs of the system and maximize the satisfaction of patients. This paper establishes an integer programming model and presents a new Lagrangian relaxation algorithm for this problem. In this algorithm, precedence constraints are relaxed by Lagrangian multipliers. In this way the relaxed problem can be decomposed into multiple stage level subproblems, each of which corresponds to a specific stage. A branch and bound algorithm is designed for solving these subproblems via developing a lower bound and dominance rules. The multipliers are then iteratively updated along a subgradient direction. Finally, the new algorithm is computationally compared with the commonly used Lagrangian relaxation algorithms which, after precedence constraints are relaxed, decompose the relaxed problem into stage level subproblems and solve the subproblems by using the dynamic programming algorithms. Numerical results show that the designed Lagrangian relaxation method produces better schedules with much smaller duality gap in acceptable computational time, especially for large-scale problems. Also, the case study shows that the application of the proposed theory results in noticeable cost saving.

1. Introduction

In the last decades, facing ever the increasing health care demand, limited government support and increasing competition, hospitals have dealt with increasing pressure of providing quality health care service while suppressing medical service costs. In a hospital environment, the operating theatre does not only represent the greatest revenue source, but also is the largest cost center and consumes a large proportion of total expenses (Lamiri, Augusto, & Xie, 2008). For these reasons, how to maintain a high quality of health care while the operational costs can be minimized becomes one of the most challenging issues for the hospitals. An effective and efficient scheduling system of operating theatres provides an appealing solution to the challenging problem (Su, Lai, Wang, Hsieh, & Lin, 2011).

The scheduling problems of operating theatres have been extensively covered in the literature. Among those studies, a hospital operating theatre generally admits two major classes of patients: elective patients, who can be planned in advance and emergency patients, who arrive randomly during the day and must be served immediately (Fei, Meskens, & Chu, 2010; Lamiri, Xie, & Zhang, 2008; Stuart & Kozan, 2012). Considering that the elective patients compose an important part of an operating theatre's capacity, in this paper, we focus on the construction of an effective schedule for the former during a certain time period.

Currently, many researchers have tried to develop efficient models and heuristic algorithms for the scheduling problems of operating theatres. A review of the research on operating room planning and scheduling, including multiple fields that are related to either the problem setting or the technical features, can be found in Cardoen, Demeulemeester, and Beliën (2010). Besides, Su et al. (2011) presented a SOMO-based approach to address the operating room scheduling problem which considered assigning an operation to the same medical specialty to minimize the total weighted idle time and the completion time of the last operation. Liu, Chu, and Wang (2011) built an open scheduling model and developed a heuristic algorithm which considered assigning

* Corresponding author.
E-mail address: bhzhou@tongji.edu.cn (B.-h. Zhou).

http://dx.doi.org/10.1016/j.cie.2016.09.003
0360-8352/

operations to the time slots available in a planning horizon. Vijayakumar, Parikh, Scott, Barnes, and Gallimore (2013) introduced a mixed-integer programming model for a multi-period, multi-resource, patient-priority-based surgical case scheduling problem and presented an efficient First Fit Descending-based heuristic. Focusing on the scheduling problem of the ophthalmology department, Devi, Rao, and Sangeetha (2012) developed a general framework of algorithms to schedule operating rooms optimally by forecasting the surgery time. Adan, Bekkers, Dellaert, Vissers, and Yu (2009) focused on a cardiothoracic surgery planning, and considered patients' stochastic durations for the stay in the Intensive Care unit (ICU) and in the Medium Care unit (MCU). They presented a mixed integer linear programming model to determine a cyclic master operation schedule. Aringhieri, Landa, Soriano, Tànfani, and Testi (2015) presented a two level meta-heuristic algorithm that solved the joint master surgical schedule and advance scheduling problem taking into account many resource and operative constraints while minimizing the total social cost of the resulting surgery schedule.

Determining an optimal schedule is a difficult problem, as many other researchers did, the considered problem is divided into two steps. Fei, Meskens, and Chu (2006) proposed the two-step approach: the first step was solved by a column-generation-based heuristic procedure. The second step was solved by a hybrid genetic algorithm, and the objective was to minimize the cost for assigning surgical cases to a set of available operating rooms during one week considering restrictions on each operating room's capacity and each surgical case's deadline. Fei et al. (2010) used the same method mentioned above, and the objectives were to maximize the utilization of the operating rooms, to minimize the overtime cost in the operating theatre, and to minimize the unexpected idle time between surgical cases. Souki and Rebai (2010) proposed a set of dispatching rule-based heuristics to solve the planning problem, and three meta-heuristics to address the scheduling problem. However, it was possible that a bad assignment of surgical cases in the first phase would influence the efficiency of the final operating program.

Other works dealt with the use of simulation for the scheduling problems of operating theatres. Everett (2002) described the design of a simulation model to provide decision support for the scheduling of patients waiting for elective surgery in the public hospital system. Bowers and Mould (2004) provided a simulation model for examining the design of orthopaedic trauma theatre sessions and the trade-off between utilization and overrunning. Testi, Tanfani, and Torre (2007) developed a three-phase simulation model for scheduling surgery rooms without increasing operating theatre time. Tànfani and Testi (2010) proposed an integrated framework for surgery department performance evaluation taking advantage both from simulation and optimization ability to support decision.

Though these studies mentioned above presented two main approaches: optimization and simulation for solving the scheduling problems of operating theatres, they did not consider the surgery process in the operating theatres which should not be ignored in real situations. The surgery process consists of three stages: the preoperative (a patient is transferred to an operating room), perioperative (the patient undergoes a surgery), and post-operative stage (the patient is moved to a recovery bed).

To solve the situations above, Pham and Klinkert (2008) proposed a new surgical case scheduling approach which used a novel extension of the job shop scheduling problem and pointed out the importance of coordinating multiple resources during any surgical step. Different conditions of elective patients were considered. The problem was formulated as a mixed integer linear programming problem and solved by CPLEX. Lamiri, Augusto, et al. (2008) proposed mixed integer linear programming based on a 3 stages hybrid flow shop scheduling problem for the scheduling problem of operating theatres. Three types of resources were considered: porters, operating rooms and recovery beds. A column generation as a decomposition approach was explored to solve the scheduling problem. Huang et al. (2012), Fei et al. (2010), Dekhici and Khaled (2010) and Yu, Yunhui, Huabo, and Jiafu (2013) regarded the surgical scheduling problem as the hybrid flow-shop scheduling problem which only contains two stages: the preoperative and perioperative stage, and only two resources were considered: beds and operating rooms. The methods they used to solve the problem above were LINGO, a hybrid genetic algorithm, a Tabu search, and a discrete particle swarm optimization algorithm, respectively.

As mentioned above, the scheduling problems of operating theatres have been attracting a lot of attentions from academia. There are numerous studies including exact techniques, heuristics and meta-heuristic algorithms. Unfortunately, these algorithms cannot provide an easy measurement on solution quality. Therefore, it is necessary to develop more effective and efficient algorithms for solving the scheduling problems of operating theatres.

Lagrangian relaxation (LR) is a decomposition and coordination approach that can yield near optimal schedules in a reasonable computational time with a quantifiable accuracy (Mao, Pan, Pang, & Chai, 2014). There has been a great deal of complex scheduling problems concentrated on LR algorithms which has emerged as a practical approach. Mao et al. (2014) proposed a novel LR approach to a hybrid flow shop scheduling problem arising from the steelmaking-continuous casting process. Zhou and Lu (2010) studied the flow shop scheduling problem and used LR to effectively minimize the total weighted completion time. Jiang and Tang (2008) focused on a re-entrant hybrid flow shop scheduling problem with the objective of minimizing the sum of weighted completion time of jobs and applied LR to solve it. Nishi, Hiranaka, and Inuiguchi (2007) proposed a successive LR method to solve flow shop scheduling problems with total weighted tardiness.

LR algorithm has been a powerful tool for solving difficult optimization problems and can be an appropriate solution method for the scheduling problems of operating theatres. Augusto, Xie, and Perdomo (2008) addressed the problem as a two stage hybrid flow shop scheduling problem with considering the cleaning time between surgeries. The postoperative stage was neglected. Three types of resources were considered: operating rooms, porter teams and recovery beds. Augusto, Xie, and Perdomo (2010) investigated the impact of allowing patient recovery in the operating room when no recovery bed was available. Three types of identical resources were considered: transporters, operating rooms and recovery beds. The problem was modelled as a 4-stage hybrid flow shop problem with blocking constraint. The 4-stages were: (i) the transport from the ward to the operating theatre, (ii) the surgery and recovery, (iii) the cleaning task and (iv) the transport from the operating theatre to the ward. The considered resources were: transporters, operating rooms and recovery beds. Nevertheless, the subproblems were solved by dynamic programming, and did not consider enough resources during the process. Moreover, the different conditions of patients were ignored by most scheduling studies.

As for the objectives, some research have made efforts to optimize a single performance criterion such as maximization of the operating room utilization and minimization of the makespan (Adan et al., 2009; Augusto et al., 2008, 2010; Devi et al., 2012; Huang et al., 2012; Lamiri, Augusto, et al., 2008; Lamiri, Xie, et al., 2008; Liu et al., 2011; Pham and Klinkert, 2008; Stuart and Kozan, 2012; Su et al., 2011; Vijayakumar et al., 2013), while many others have included several performance criteria in their studies (Fei et al., 2010; Meskens, Duvivier, & Hanset, 2013; Souki & Rebai, 2010). Due to the costs of the operating theatres are very important to the hospitals, the minimum related costs of the sys-