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Intuitionistic multiplicative analytic hierarchy process in group decision making

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ABSTRACT

Analytic Hierarchy Process (AHP), which analyzes complex decisions by organizing the problems into a multilayer hierarchic structure, is a simple yet popular decision technique used extensively in every decision field. But it is inadequate to handle the uncertain decision making problems. Taking advantage of intuitionistic multiplicative information in portraying the vagueness of problems with Saaty's 1/9–9 scale, in this paper, we extend the intuitionistic multiplicative information into AHP to enhance the ability of AHP in tackling various decision making problems. We first verify that the intuitionistic multiplicative weighted geometric aggregation (IMWGA) operator has desirable characteristics to guarantee that the overall intuitionistic multiplicative preference relation (IMPR) is consistent when all individual IMPRs are consistent. Then, we provide a whole procedure of intuitionistic multiplicative analytic hierarchy process for solving group decision making problems, including adjusting the individual IMPRs, aggregating the individual IMPRs and deriving the priorities from the overall IMPR. Finally, we present an example concerning the performance assessments of the hydropower stations to illustrate the effectiveness and applicability of the group intuitionistic multiplicative analytic hierarchy process.

1. Introduction

Decision theory, which aims to identify a desirable alternative according to the descriptive information of decision makers (DMs), has widespread use in every field of our modern life. Due to the incomplete information and the uncertainties of the problems, the DMs cannot accurately quantify the characteristics of alternatives. Till now, there are basically two types of preference relations which are used to express the DMs' preferences on alternatives: fuzzy preference relations and multiplicative preference relations (Herrera, Herrera-Viedma, & Chiclana, 2001; Orlovsky, 1978; Saaty, 1980; Xu, 2007). As for the multiplicative preference relations, Saaty (1980) introduced the 1/9–9 scale to represent the characteristics of the objects. With this scale, the elements of the multiplicative preference relations are all in [1/9, 9]. However, Saaty's 1/9–9 scale can only depict the affirmative preference information but ignores the negative and hesitative preference information over the objects. To overcome this drawback, Xia, Xu, and Liao (2013) defined the intuitionistic multiplicative set (IMS), which includes a membership degree, a non-membership degree and a hesitancy degree, whose values vary between 1/9 and 9, to describe the DMs' preferences more comprehensively.

Analytic Hierarchy Process (AHP) introduced by Saaty (1977, 1990), is one of the most significant methods to deal with the decision making problems. By organizing the objectives, the criteria (sub-criteria), and the alternatives into a multilayer hierarchical structure, AHP can analyze complex decision making problems efficiently. AHP obtains the priorities of each criterion and synthesizes the scores for each alternative on different criteria. The details of AHP involve the following steps:

(1) Analyzing the problem and constructing the hierarchical structure;
(2) Determining the multiplicative preference relations by pairwise comparisons of the criteria, and provide the decision values of the alternatives with respect to each criterion;
(3) Deriving the priorities of the criteria from the multiplicative preference relations;
(4) Aggregating the comprehensive values for each alternative and ranking all alternatives.

AHP is a simple yet popular decision making technique which has been widely used in the fields of business (Angelou & Economides, 2009; Arbel & Orgler, 1990; Chen & Wang, 2010; Smyth & Lecoeuvre, 2015), industry (Al-Oqla, Sapuan, Ishak, & Nuraini, 2015; Chen & Wang, 2010; Yang, Chuang, & Huang, 2009), healthcare (Ajami & Ketabi, 2012; Case, O'Leary, Kim, Tinetti, & Fried, 2015; Danner et al., 2011), and so on. Even though

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AHP has a strong ability in handling general decision making problems, it has also the limitations in applying the AHP to the uncertain decision making problems. The traditional AHP method, which is described by Saaty's 1–9 scale, lacks the practicability in some cases due to the fact that the DMs may not provide the accurate numbers to represent their opinions with respect to the uncertainty and vagueness of the problems. To improve this situation, some scholars have combined the fuzzy set theory (Zadeh, 1965) and the AHP by introducing trapezoidal fuzzy numbers (Buckley, 1985) and triangular fuzzy numbers (Van Laarhoven & Pedrycz, 1983) into the AHP. Later on, some studies have been conducted to derive the priorities (Boender, de Graan, & Lootsma, 1989; Chou, Sun, & Yen, 2012) and developed the corresponding decision making methods (Fan, 2016; Gao, Li, & Zhang, 2015; Jaiswal, Ghosh, Lohani, & Thomas, 2015; Zhang, Bouras, Ouzrout, & Sekhari, 2014; Zhu, Jing, & Chang, 1999; Zyoud, Kaufmann, Shaheen, Samhan, & Fuchs-Hanusch, 2016) within the context of fuzzy AHP. The fuzzy AHP can only be used to solve the fuzzy decision making problems with the preference information of symmetrical distribution, however, we usually need to use the preference information with unbalanced distribution to deal with the decision making problems, just as the law of diminishing marginal utility in economics mentioned by Xia et al. (2013). Thus, in order to enhance the applicability of the AHP with Saaty's 1–9 scale, it is urgent to do some work in extending the traditional AHP to the unbalanced distribution situation. As the IMS can describe the objects more comprehensively by providing the information of superiority, inferiority and hesitation, this paper aims to integrate the IMS into AHP so as to derive much more reasonable decision results in practical decision making problems. Meanwhile, considering that a majority of decision processes require multiple stakeholders, the focus of our work is to tackle group decision making problems.

Based on the above analysis, we organize the paper as follows: Section 2 reviews some fundamental knowledge about the intuitionistic multiplicative number (IMN) and the intuitionistic multiplicative preference relation (IMPR). Based on the consistency of the IMPR, Section 3 shows that the intuitionistic multiplicative weighted geometric aggregation (IMWGA) operator has desirable characteristics to guarantee that the overall IMPR is also consistent (or acceptably consistent) when all individual IMPRs are consistent (or acceptably consistent). Some properties of the IMWGA operator are also given in this section. Section 4 provides a whole procedure of group intuitionistic multiplicative analytic hierarchy process (GIMAHP), including adjusting the individual IMPRs, aggregating the individual IMPRs and deriving the priorities from the overall IMPR. An example concerning the performance assessments of the hydropower stations is presented in Section 5 to illustrate the applicability of the GIMAHP. Finally, some conclusions are listed in Section 6.

2. Preliminaries

Here we review some elementary knowledge about intuitionistic multiplicative number (IMN) and intuitionistic multiplicative preference relation (IMPR).

2.1. IMNs

As the basic component of IMPR, IMN (Xia et al., 2013) expressed as $\alpha = (\rho_\alpha, \sigma_\alpha)$ is an effective tool to depict the superiority and the inferiority of an objective, where ρ_α and σ_α, which both belong to $[1/9, 9]$, respectively indicate the membership degree and the non-membership degree. The hesitation degree of α is defined by $\tau_\alpha = 1/(\rho_\alpha \sigma_\alpha)$. Some operational laws and the ranking method for IMNs can be given below:

Definition 2.1 Xia et al. (2013). Let $\alpha = (\rho_\alpha, \sigma_\alpha)$, $\alpha_1 = (\rho_{\alpha_1}, \sigma_{\alpha_1})$ and $\alpha_2 = (\rho_{\alpha_2}, \sigma_{\alpha_2})$ be three IMNs, and $\lambda > 0$, then

(1) $\alpha_1 \oplus \alpha_2 = \left(\frac{(1+2\rho_{\alpha_1})(1+2\rho_{\alpha_2})-1}{2}, \frac{2\sigma_{\alpha_1}\sigma_{\alpha_2}}{(2+\sigma_{\alpha_1})(2+\sigma_{\alpha_2})-\sigma_{\alpha_1}\sigma_{\alpha_2}}\right)$;

(2) $\alpha_1 \otimes \alpha_2 = \left(\frac{2\rho_{\alpha_1}\rho_{\alpha_2}}{(2+\rho_{\alpha_1})(2+\rho_{\alpha_2})-\rho_{\alpha_1}\rho_{\alpha_2}}, \frac{(1+2\sigma_{\alpha_1})(1+2\sigma_{\alpha_2})-1}{2}\right)$;

(3) $\lambda\alpha = \left(\frac{(1+2\rho_\alpha)^\lambda-1}{2}, \frac{2\sigma_\alpha^\lambda}{(2+\sigma_\alpha)^\lambda-\sigma_\alpha^\lambda}\right)$;

(4) $\alpha^\lambda = \left(\frac{2\rho_\alpha^\lambda}{(2+\rho_\alpha)^\lambda-\rho_\alpha^\lambda}, \frac{(1+2\sigma_\alpha)^\lambda-1}{2}\right)$.

Definition 2.2 Xia et al. (2013). The expressions $s(\alpha) = \rho_\alpha/\sigma_\alpha$ and $h(\alpha) = \rho_\alpha\sigma_\alpha$ are respectively the score function and the accuracy function of an IMN $\alpha = (\rho_\alpha, \sigma_\alpha)$. Then for two IMNs α_1 and α_2, we have:

(1) If $s(\alpha_1) > s(\alpha_2)$, then $\alpha_1 > \alpha_2$;
(2) If $s(\alpha_1) = s(\alpha_2)$, then
  (a) If $h(\alpha_1) > h(\alpha_2)$, then $\alpha_1 > \alpha_2$;
  (b) If $h(\alpha_1) = h(\alpha_2)$, then $\alpha_1 = \alpha_2$.

The aggregation operators are useful to integrate IMNs in the decision making process. Here we introduce a commonly used aggregation technique for IMNs, i.e., the intuitionistic multiplicative weighted operator. Moreover, it should be noted that the $\oplus$ operation is associative, that is, $\alpha \oplus (\beta \oplus \gamma) = (\alpha \oplus \beta) \oplus \gamma$ for any three IMNs α, β and γ.

Definition 2.3 Xia et al. (2013). Let $\alpha_1, \ldots, \alpha_n$ be a collection of IMNs, then an intuitionistic multiplicative weighted averaging (IMWA) operator is expressed as:

$$IMWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = \mathop{\oplus}_{i=1}^{n} (\omega_i \alpha_i) = \left(\frac{\prod_{i=1}^{n}(1+2\rho_{\alpha_i})^{\omega_i}-1}{2}, \frac{2\prod_{i=1}^{n}\sigma_{\alpha_i}^{\omega_i}}{\prod_{i=1}^{n}(2+\sigma_{\alpha_i})^{\omega_i}-\prod_{i=1}^{n}\sigma_{\alpha_i}^{\omega_i}}\right) \tag{2.1}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight vector of α_i with $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n}\omega_i = 1$.

2.2. IMPRs

IMPR is obtained by comparing objects in pairs to judge the preferred one, and the results of the pairwise comparisons are expressed by IMNs. More specifically, for a set of objects $O = \{o_1, o_2, \ldots, o_n\}$, Xia et al. (2013) provided the IMPR $A = (\alpha_{ij})_{n\times n}$ to compare one object with another, which is measured by Saaty's $1/9-9$ scale, where $\alpha_{ij} = (\rho_{\alpha_{ij}}, \sigma_{\alpha_{ij}})$ is an IMN. $\rho_{\alpha_{ij}}$ and $\sigma_{\alpha_{ij}}$ respectively indicate the degree to which o_i is preferred and not preferred to o_j with the conditions $\rho_{\alpha_{ij}} = \sigma_{\alpha_{ji}}$, $\sigma_{\alpha_{ij}} = \rho_{\alpha_{ji}}$, $\rho_{\alpha_{ii}} = \sigma_{\alpha_{ii}} = 1$, $0 < \rho_{\alpha_{ij}}\sigma_{\alpha_{ij}} \leqslant 1$ and $1/9 \leqslant \rho_{\alpha_{ij}}, \sigma_{\alpha_{ij}} \leqslant 9$. By the expression $\tau_\alpha = 1/(\rho_\alpha\sigma_\alpha)$, the hesitancy degree that o_i is preferred to o_j is located in the interval $[1, 81]$.

Generally, the consistency of IMPR is significant in the decision making process. One definition about the consistency of IMPR was given as (Jiang, Xu, & Yu, 2015):

Definition 2.4 Jiang et al. (2015). Let $A = (\alpha_{ij})_{n\times n}$ be an IMPR with $\alpha_{ij} = (\rho_{\alpha_{ij}}, \sigma_{\alpha_{ij}})$, $C = (c_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ be two MPRs, where