Engineering Analysis with Boundary Elements 36 (2012) 1040–1048

Contents lists available at SciVerse ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

An analysis of the convection–diffusion problems using meshless and meshbased methods

Xue-Hong Wu [a,]*, Zhi-Juan Chang [a], Yan-Li Lu [a], Wen-Quan Tao [b], Sheng-Ping Shen [b]

[a] *Zhengzhou University of Light Industry, 450002 Zhengzhou, Henan, China*
[b] *Xi'an Jiaotong University, 710049 Xi'an, Shannxi, China*

ARTICLE INFO

Article history:
Received 15 December 2010
Accepted 28 July 2011
Available online 19 December 2011

Keywords:
MLPG
SUPG
MLS
Convection–diffusion
Upwind scheme

ABSTRACT

The numerical solution of the convection–diffusion equation represents a very important issue in many numerical methods that need some artificial methods to obtain stable and accurate solutions. In this article, a meshless method based on the local Petrov–Galerkin method is applied to solve this equation. The essential boundary condition is enforced by the transformation method, and the MLS method is used for the interpolation schemes. The streamline upwind Petrov–Galerkin (SUPG) scheme is developed to employ on the present meshless method to overcome the influence of false diffusion. In order to validate the stability and accuracy of the present method, the model is used to solve two different cases and the results of the present method are compared with the results of the upwind scheme of the MLPG method and the high order upwind scheme (QUICK) of the finite volume method. The computational results show that fairly accurate solutions can be obtained for high Peclet number and the SUPG scheme can very well eliminate the influence of false diffusion.

1. Introduction

In spite of the great success of the finite volume method and finite element method as effective numerical tools for solving fluid flow and heat transfer problems, there are also some shortcomings. These methods depend strongly on mesh properties; developing good quality mesh is a time-consuming and burdensome task, particularly in 3D; they also suffer some difficulties in solving problems of shear band formation, large deformation, etc.; these difficulties can be overcome by the so-called meshless method, which has received much attention and achieved remarkable applications over the past decade.

In the early stage of the development of meshless methods, they were usually used in computational mechanics. A number of meshless methods have been applied by different authors to compute heat transfer and fluid dynamics. Cleary and Monaghan [1] applied the smooth particle hydrodynamics (SPH) method to solve unsteady-state heat conduction problems. At present, the SPH method has been employed widely on the incompressible flow problems [2–4] and free surface flow problems [5–7]. Singh et al. [8–10] applied the element free Galerkin (EFG) method to solve 2-D and 3-D heat conduction problems. Fries and Matthies [11,12] developed the coupled method of EFG and FEM to compute incompressible flow problems. Oñate et al. [13] employed the finite point method (FPM) to compute convection–diffusion problems. Gunther et al. [14] used the reproducing kernel particle method (RKPM) to solve viscous, compressible flow problems. Lin and Atluri [15,16] proposed two kinds of upwind schemes for the meshless local Petrov–Galerkin (MLPG) method to solve convection–diffusion problems and incompressible flow problems. Liu and his collaborators [17] developed a meshfree weak–strong (MWS) method and used it to solve 2-D laminar natural convection problems. Shu et al. [18] proposed the RBF-DQ method to compute incompressible flow. Liu et al. [19] developed a weighted least-squares (MWLS) method to solve steady and unsteady-state heat conduction problems. Liu and Tan [20] solved coupled radiative and conductive heat transfer problems using the MLPG collocation method. Wu et al. [21] and Wu and Tao [22] applied the MLPG method to solve heat conduction problems for irregular domains encountered in engineering. They compared their results with those of the finite volume method (FVM) and their results showed that the computational precision of the MLPG method was much better than that of FVM. Arefmanesh et al. [23] applied the MLPG method to compute non-isothermal fluid flow problems with the vorticity-stream function method and the unity function applied as a weight function. Mohammadi [24] constructed a new upwind scheme to compute incompressible flow problems with the vorticity-stream function method; in his work, the Heaviside step function was used as the test function and radial basis function (RBF) interpolation was

* Corresponding author.
E-mail address: wuxh1212@yahoo.com.cn (X.-H. Wu).

0955-7997/$ - see front matter
doi:10.1016/j.enganabound.2011.07.014

employed on the shape function and its derivatives construction. The results showed that this method had very good accuracy. The approaches based on the vorticity-stream function method can satisfy the incompressible mass conservation condition automatically, but can not be directly extended to solve 3-D problems. Wu et al. [25,26] applied the node-based smoothed point interpolation method (NS-PIM) to solve 3-D heat transfer and the thermal analysis of the process plasma spraying.

From the above brief review on applications of meshless methods in computational fluid dynamics and heat transfer, we can see that previous researchers have focused mainly on using the upwind scheme or the method of adding a stability term to overcome the influence of the non-linear convection term. The meshless methods based on the global weak form (GWF), such as EFG, RKPM and NS-PIM, etc., are not "truly meshless" methods because they require a background cell for numerical integration over the global domain in the stiffness matrix system. However, Atluri and his colleagues proposed the MLPG method, which was based on the local weak form (LWF). This method is the "truly meshless" because it does not require a mesh for either interpolation or numerical integration. A wide range of problems in computational mechanics have been investigated by Atluri and his co-authors [27]. Simultaneity, Lin and Atluri [15,16] also developed two upwind schemes (US1,US2) to deal with the convection term for flow problems; their results showed that the second upwind scheme was better, but when this method was used to handle incompressible flow problems for high Reynolds number it suffered from divergence problems. It is necessary to build some stability and a high precise upwind scheme to deal with the convection term in the MLPG method.

The purpose of this paper is to introduce a streamline upwind Petrov–Galerkin method into the MLPG method for flow problems. The new scheme is applied in convection–diffusion problems to verify its stability and accuracy. The results of the present method are also compared with the results of the upwind scheme proposed by Lin and Atluri and the high order scheme using FVM.

2. MLPG method for the convection–diffusion equation

The two-dimensional, steady-state convection–diffusion equation in the entire domain Ω and boundary conditions can be written in the following form:

$$u_j \frac{\partial T}{\partial x_j} = \frac{1}{Pe}\frac{\partial^2 T}{\partial {x_j}^2} + Q \quad (j=1,2) \text{ in } \Omega \tag{1}$$

where the Peclet number is defined as

$$Pe = \frac{u_{ref} L_{ref}}{a} \tag{2}$$

the Dirichlet boundary condition

$$T = \overline{T}_1 \text{ on } \Gamma_u \tag{3}$$

the Neumann boundary condition

$$-\lambda \frac{\partial T}{\partial x_j} n_j = \overline{q}_1 \text{ on } \Gamma_t \tag{4}$$

where T represents the temperature, $\overline{T}_1$ is the given boundary temperature, λ is the thermal conductivity, Q is the source term, n_j is the outward unit vector to Γ, $\overline{q}_1$ is the given boundary heat flux, u_j is the velocity, and Γ_t and Γ_u are subsets of Γ satisfying $\Gamma_t \cap \Gamma_u = \emptyset$ and $\Gamma_t \cup \Gamma_u = \Gamma$.

To satisfy Eq. (1) in a local sub-domain Ω_x, the weighted integral form of Eq. (1) is given as

$$\int_{\Omega_x} \left(u_j \frac{\partial T}{\partial x_j} - \frac{1}{Pe}\frac{\partial^2 T}{\partial x_j^2} - Q \right) w \, d\Omega_x = 0 \tag{5}$$

To reduce the order of the required differentiability on T, we can integrate Eq. (5) by parts

$$\int_{\Omega_x} \frac{\partial^2 T}{\partial {x_j}^2} w \, d\Omega_x = \int_{\Omega_x} \left[\frac{\partial}{\partial x_j}\left(\frac{\partial T}{\partial x_j} w \right) - \frac{\partial T}{\partial x_j}\frac{\partial w}{\partial x_j} \right] d\Omega_x \tag{6}$$

By using the Gauss theorem

$$\int_{\Omega_x} \frac{\partial}{\partial x_j}\left(\frac{\partial T}{\partial x_j} w \right) d\Omega_x = \int_{\Gamma_s} \left(\frac{\partial T}{\partial x_j} w \right) n_j \, d\Gamma \tag{7}$$

we obtain the following local weak formulation equation:

$$\int_{\Omega_x} \left(u_j \frac{\partial T}{\partial x_j} w + \frac{1}{Pe}\frac{\partial T}{\partial x_j}\frac{\partial w}{\partial x_j} - Qw \right) d\Omega_x - \int_{\Gamma_s} \frac{1}{Pe}\frac{\partial T}{\partial x_j} n_j w \, d\Gamma = 0 \tag{8}$$

Γ_s is the sub-domain boundary, the boundary Γ_s of the local sub-domain includes three parts (see Fig. 1), Γ_{sI}, Γ_{su} and Γ_{st}. Γ_{sI} is the part of the sub-domain boundary included in the global domain, $\Gamma_{su} = \Gamma_s \cup \Gamma_u$, and $\Gamma_{st} = \Gamma_s \cup \Gamma_t$.

Substituting Eq. (4) into Eq. (8), results in the following equation:

$$\int_{\Omega_{\mathbf{x}}} \left(u_j \frac{\partial T}{\partial x_j} w + \frac{1}{Pe}\frac{\partial T}{\partial x_j}\frac{\partial w}{\partial x_j} \right) d\Omega - \int_{\Gamma_{sI}} \frac{1}{Pe}\frac{\partial T}{\partial x_j} n_j w \, d\Gamma - \int_{\Gamma_{su}} \frac{1}{Pe}\frac{\partial T}{\partial x_j} n_j w \, d\Gamma + \int_{\Gamma_{st}} \frac{1}{\lambda Pe} \overline{q}_1 w \, d\Gamma - \int_{\Omega_{\mathbf{x}}} Qw \, d\Omega = 0 \tag{9}$$

To obtain the discretized equation of each sub-domain, the unknown function can be approximation using the moving least square (MLS) method

$$T^h(\mathbf{x}) = \mathbf{\Phi}^{\mathbf{T}}(\mathbf{x})\hat{\mathbf{T}} = \sum_{I=1}^{N} \mathbf{\Phi}_I(\mathbf{x})\hat{T}_I \tag{10}$$

where $\hat{\mathbf{T}}$ represent the fictitious nodal value, but not the value of unknown function. The characteristics of MLS have been widely discussed in the literature [28,29] and will not be restated here. Substitution of Eq. (10) into the Eq. (9) for all the nodes, yields the following discretized system of linear equations:

$$\sum_{J=1}^{M} \int_{\Omega_{\mathbf{x}}} \left(u_j \partial \mathbf{\Phi}^J \frac{\hat{\mathbf{T}}^J}{\partial x_j} w_I + \frac{1}{Pe} \partial \mathbf{\Phi}^J \frac{\hat{\mathbf{T}}^J}{\partial x_j}\frac{\partial w_I}{\partial x_j} \right) d\Omega - \sum_{J=1}^{M} \int_{\Gamma_{sI}} \frac{1}{Pe} \partial \mathbf{\Phi}^J \frac{\hat{\mathbf{T}}^J}{\partial x_j} n_j w_I \, d\Gamma - \sum_{J=1}^{M} \int_{\Gamma_{su}} \frac{1}{Pe} \partial \mathbf{\Phi}^J \frac{\hat{\mathbf{T}}^J}{\partial x_j} n_j w_I \, d\Gamma = -\int_{\Gamma st} \frac{1}{\lambda Pe} \overline{q}_1 w_I \, d\Gamma + \int_{\Omega_{\mathbf{x}}} Q w_I \, d\Omega \tag{11}$$

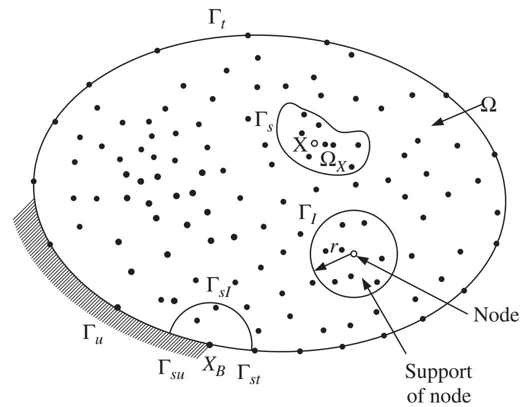

Fig. 1. Schematics of the sub-domain.