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Holling–Tanner model with Beddington–DeAngelis functional response and time delay introducing harvesting

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Highlights

- Holling–Tanner model with Beddington–DeAngelis Functional Response is analyzed.
- Harvesting on prey and Gestational delay of predator into the system are introduced.
- The direction and stability of Hopf bifurcation are established.
- A profit function with the help of bionomic equilibrium is calculated.
- Numerical simulations with graphical are included to verify the analytic results.

Abstract

The paper is formulated with the Holling–Tanner prey–predator model with Beddington–DeAngelis functional response including prey harvesting. Gestational time delay of predator and the dynamic stability of time delay preventing system are incorporated into the system of our paper. The equilibria of the proposed system are determined and the existence of interior equilibrium point for the proposed system is described. Local stability of the system with the magnitude of time delay at the interior equilibrium point is discussed. Thereafter, the direction and the stability of Hopf bifurcation are established with the help of normal theory and center manifold theorem. Furthermore, profit function is calculated with the help of bionomic equilibrium and it is optimized using optimal control. Finally, some numerical simulations are introduced to verify the validity of analytic results of our proposed model.

Keywords: Holling–Tanner prey–predator model; Harvesting; Time delay; Local stability; Hopf bifurcation

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1. Introduction

The relationship between prey and predator is natural phenomenon for universal existence in our ecological system. Interactions of prey and predator are one of the common and well known factors in ecological systems. This ecological system is one of the important fields in the study of mathematical biology. In the 1920s, Vito Volterra described whether it would be possible to clarify the fluctuations that had been noticed in the fish population of the Adriatic sea fluctuations that were of great concern to fishermen in time of low fish populations. Volterra [20] formulated the model which was treated as the Lotka–Volterra model because Lotka [11] generated a similar model in view of other context about the same time, based on the assumptions that fish and shark were in a prey–predator relationship. After that so many researchers have shown their attention on this matter. The classical experimental result on species in competition is the principle of competitive exclusion of Gause [5], that two species competing for the same resource cannot coexist. Harvesting of fisheries is an important area of study in fishery modeling. Mathematical modeling in harvesting of such fisheries was studied by Clark [2], Das et al. [3], Roy and Roy [17]. Time delay has an important role in biological population. Differential equation with time delay has received great attention in research of recent years among theoretical and mathematical ecologists. Although, study of time delay can have very complex impact on the dynamics of a system, for example, it can cause the loss of stability; can induce various oscillations and periodic solutions. Few researchers have shown their interest in this direction such as Gopalsamy [6]; Jana et al. [8]; Roy and Roy [16]; Sakera and Alzabutb [19]. In different ecological systems, interaction between predator and prey is also different. Due to this reason functional response is changed. Beddington–DeAngelis functional response is most important in biological interaction. This functional response was constructed by Beddington [1] and DeAngelis et al. [4]. Lu and Liu [12] described a prey predator model with modified Holling–Tanner functional response with time delay. Pal and Mandal [13] modified Leslie–Gower predator–prey model with Beddington–DeAngelis functional response with time delay.

In ecology, predation is a biological interaction where predator feeds on its prey. At any period t, let the prey population be denoted by x and predator population be denoted by y in certain ecological system. In particular case, the prey population is so large than the predator population, then we consider a particular type of prey–predator model which plays a special role in view of interesting dynamics and it possesses the Holling–Tanner predator–prey system [21]. Then the Holling–Tanner prey–predator model is defined as follows:

$$\left.\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\psi(x,y)y \\ \frac{dy}{dt} &= \beta y\left(1-\frac{y}{\gamma x}\right) \end{aligned}\right\} \tag{1}$$

with initial conditions $x(0) \geq 0$, $y(0) \geq 0$ and $\gamma > 0$. And r be the intrinsic growth rate of prey population and $K(> 0)$ be the carrying capacity of prey in the biomass. The biotic potential of the predator population is denoted by β and $\frac{1}{\gamma}$ be the amount of prey required to support a predator at equilibrium. The function $\psi(x, y)$ denotes the predator response function.

In Southern Ocean, one species, the Antarctic krill (Euphausia superba) makes up an estimated biomass. Of this, over half is eaten by whales, seals, penguins, squid and fish each year, and is replaced by growth and reproduction. Considering this phenomenon, we design the Holling–Tanner prey–predator model with Beddington–DeAngelis functional response in our paper and consider $\psi(x, y) = \frac{\alpha x}{a+bx+my}$ where α denotes the maximal super predator per capita consumptions rate, i.e., the maximum number of predator population can be eaten by a super predator in each time unit and $a, b, m > 0$. Since most of the krill is used for aquaculture and aquarium feeds, as bait in sport fishing, or in the pharmaceutical industry, for this reason, we introduce the harvesting effort $h(t)$ on prey population. Then the system of Eq. (1) reduces to

$$\left.\begin{aligned} \frac{dx}{dt} &= rx\left(1-\frac{x}{K}\right)-\frac{\alpha xy}{a+bx+my}-h(t) \\ \frac{dy}{dt} &= y\left(\beta-n\frac{y}{x}\right) \end{aligned}\right\} \tag{2}$$

where $n = \frac{\beta}{\gamma}$, $x(0) \geq 0$, $y(0) \geq 0$.