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A semi-discrete central scheme for incompressible multiphase flow in porous media in several space dimensions

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Abstract

In this work we present a Godunov-type semi-discrete central scheme for systems of conservation laws which allows for spatial heterogeneity of the storage coefficient, say, the porosity field. This scheme is used in the composition of a sequential splitting algorithm for simulating incompressible multiphase flow within rigid porous media, with both permeability and porosity fields heterogeneous. The proposed methodology composes a fundamental block in the simulation of complex flows in porous media, where the compressibility effects may be included. Numerical tests are presented to illustrate the accuracy of the proposed method in problems that simulate immiscible three-phase flow in heterogeneous porous media in two and three space dimensions.
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1. Introduction

In porous media applications, flow and transport are strongly influenced by spatial variations in permeability and porosity fields [14]. The dynamics of multiphase flow in porous media involves highly complex physics associated with different patterns of fluid mixing and finger growth. Such complex behavior is strongly dictated by heterogeneity in the rock properties acting in conjunction with fluid instabilities induced by unfavorable viscosity ratio [48]. It is often considered that heterogeneity is solely manifested in the permeability field and that variations in the porosity field are negligible [28,13]. However, in various geomechanical applications, such as the land subsidence problem, or in problems where solid matrix dissolution is considered, porosity is related with permeability and changes not only in space, but also in time [47,48].

In general, mathematical models of porous media flow are composed by systems of partial differential equations that are strongly nonlinearly coupled. In [20,4,32,50] nonlinear differential systems that model two-phase flow in porous media are written in a fractional flow formulation, i.e. in terms of a saturation and a global pressure. The three-phase case (e.g., water, oil and gas) is studied, for example, in [21], where the authors discuss various formulations of

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the governing equations including phase, global and pseudo-global pressure-saturation formulations. In this case, the relative permeability and capillary pressure curves are far more complex than the corresponding two-phase curves and additional hypothesis must be assumed to derive a global pressure-saturation form for the three-phase flow. The idea of separating the equations in two sets with a very different nature is motivated by petroleum reservoir simulation, where efficient numerical methods can be devised to take advantage of many physical properties inherent in the flow equations [11,50,32]. In this paper we consider the multiphase flow of incompressible phases within a rigid, but highly heterogeneous, porous medium by using the fractional flow formulation [50]. Despite being simple, this model allow us to study important aspects of porous media flow that are present in more complex models, focusing on the development of efficient and robust numerical schemes that can be used as blocks for the simulation of real problems. If the diffusion effects can be neglected, the mass conservation equations of the phases can be written as a nonlinear hyperbolic system with porosity appearing in the storativity term [11] while the total flux (velocity field) and pressure are found by the solution of a first order system of elliptic nature [53].

From the numerical point-of-view, the accurate representation of these phenomena requires the development of computational schemes that must be able to deal with discontinuous and heterogeneous coefficients, while maintaining local conservation properties. Thus, the development of new classes of locally conservative computational schemes capable of capturing in an accurate fashion the effects of spatial variability in the porous medium properties is a challenging issue. In general, the transport dominates the entire process. Hence it is important to obtain good approximate velocities. The classical Galerkin method with $H^1(\Omega)$ continuous nodal basis functions enforces conservation of mass only approximately. It motivates the use of mixed finite elements, based on simultaneous approximation of the pressure and velocity. The use of conform and stable $H(\mathrm{div}, \Omega) \times L^2(\Omega)$ subspaces such as the Raviart–Thomas spaces [55,12] or the Brezzi–Douglas–Marini spaces [16] provides locally conservative schemes that mimic the local physics by approximating heterogeneous velocity fields using continuous interpolation across element edges for the normal flux [32,31,26,27]. The solution of elliptic problems by locally conservative methods which handle discontinuous coefficients on general irregular grids can be found, for example, in [35,19,32,27,5]. In this paper we consider only orthogonal meshes, in two or three dimensions, and adopt the hybridized version of the dual mixed finite element method with the lowest order index Raviart–Thomas–Nédélec space [55,51]. This hybrid formulation is equivalent to the conventional mixed method, but more efficient [32,50].

Regarding the transport problem, saturation entropy solutions admit non-smooth composite waves (shocks and rarefaction waves) and the development of robust and accurate numerical methods for hyperbolic problems is still a challenging issue that has been extensively investigated. Several stable and robust first-order methods have been proposed and extended to higher-order methods [30,33,23,36,43,37,59]. We are interested in Godunov-type schemes, that are projection–evolution methods that can be embedded in the so-called REA (Reconstruct Evolve Average) algorithms. Depending on the projection step, two kinds of Godunov schemes can be defined [39]—central and upwind. The great advantage of Godunov-type central schemes is that, in contrast to the Godunov-type upwind schemes, no decomposition or Riemann solvers are involved. Thus, among these methods, we highlight the higher-order central finite volume methods [52,40,39,38,24], which provide an attractive class of alternative robust, efficient and universal Riemann-solver-free methods that have been successfully used for solving nonlinear conservation laws. Such schemes consist of an alternative class of efficient and simple methods usually built by using the concept of staggered mesh to evolve the smooth parts of the solution, avoiding the need of solving the Riemann problem at cell interfaces for computation of the numerical flux [56].

The forerunner of such methods is the first-order Lax–Friedrichs (LxF) scheme [41] that has limited acceptance because of its excessive numerical dissipation, later reduced in the high-order extension by Nessyahu and Tadmor (NT) [52]. The class of higher-order methods achieves higher convergence rates by reconstructing a non-oscillatory piecewise polynomial interpolation on each cell from the original average values, followed by an evolution operator to the next time level and finally projecting back to a piecewise constant solution onto the original grid. In the NT scheme, the first-order piecewise constant functions of the LxF scheme are replaced by second-order piecewise linear MUSCL (Monotone Upstream-centered Schemes for Conservation Laws) interpolants [58]. The LxF and NT schemes exhibit numerical dissipation $\mathcal{O}(\Delta x^{2r}/\Delta t)$, where r is the formal order of the method ($r = 1$ for LxF and $r = 2$ for NT). Therefore, despite having higher precision, the NT scheme does not overcome the difficulties that arise when small time steps are used, normally imposed by CFL-type restrictions [52,40,24]. One way to overcome that difficulty is to use semi-discrete formulations (the LxF and NT methods do not admit semi-discrete formulations), that can combine high-resolution, non-oscillatory spacial discretization with high-order, large stepsize Ordinary Differential

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