Original articles

Shape effects on herd behavior in ecological interacting population models

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Highlights

- Several dynamical systems modeling two-populations interactions gathering in herds.
- Interaction occurs on the perimeter in $2D$.
- Interaction occurs on the total surface area for populations living in $3D$.
- Here we even accommodate the model for herds that assume fractal shapes.
- Populations symbiosis, competition and predator–prey are considered.
- Stable solution is independent of the shape of the herd for competition.

Abstract

In this paper, we introduce several dynamical systems modeling two-populations interactions. The main idea is to assume that the individuals of one of the populations gather together in herds, thus possess a social behavior, while individuals of the second population show a more individualistic attitude. We model the fact that the interaction between the two populations occurs mainly through the perimeter of the herd in a $2D$ space or through the total surface area for populations that live in a $3D$ space. This idea has already been explored earlier, but here we even accommodate the model for herds that assume fractal shapes. We account for all types of the populations intermingling: symbiosis, competition and predator–prey interactions. In the cases of obligated mutualism for the individualistic population and of competition, the stable solution attained by the populations is independent of the shape of the herd.
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1. Introduction

Nonlinear models for populations interactions have been considered under various assumptions, starting from the fact that prey consumption is limited by handling and digesting it, [9,16,17,23], or the environment carrying capacity

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may depend on the prey size, [20]. The ratio-dependent models have caused some discussion among specialists, [1–3, 6,18]. For a broader presentation of these concepts, see part I of [22].

In particular models for the interactions of populations that exhibit a social kind of behavior in various situations are arousing interest among the scientific community, [13]. In particular, the recently proposed investigations on herd behavior start from [5,4]. These ideas have been resumed and extended in [10] and further explored in [8, 14], with an illustration of a more complex spatial situation in [25]. The basic assumption consists in stating that the prey population gathers in communities wandering in search of pastures, while the predators follow them. The individualistic hunt occurs in such a way that the predator aims at one of the closest individuals in the herd, that generally occupies a position on the boundary of the herd, and attacks it. Therefore the basic assumption in [5,4,13] is that the population interaction still occurs on a 1–1 basis, one prey and one predator, but the involved prey belongs only to the portion of the herd that has a position on its outskirts. This implies that if the prey distribute homogeneously, the set of the individuals on the border is proportional to the herd perimeter, which in turn is related to the square root of the herd area, and subsequently to the square root of the population size. This however is questionable in real life situations, in which the herd shape may not be a "nice" figure, but rather a complicated two or three dimensional object, if we include also fish schools in the aquatic realm or swarms of insects or birds in the air. In this paper we relax the simplifying assumption of the square root and allow for a generic exponent α to express the perimeter length of the herd. Note that an extension of these concepts to the more general case of ecoepidemic situations, i.e. interacting populations subject additionally to a transmissible disease, has been considered in [24].

The main result shows that in the case of the obligated mutualism for the individualistic population and of the competition case, the stable solutions attained by the populations is independent of the shape of the herd, which is represented by the value of the parameter α in the system (1).

The presentation follows the possible types of populations intermingling. Specifically, in the paper, we start by analyzing the predator–prey situation, in Section 2, then we move to the symbiotic case, in Section 3, and finally to the competition case, Section 4. A sensitivity analysis in terms of the shape parameter α is performed for the predator–prey model. A final discussion concludes the paper.

2. The predator–prey model

We present a modified predator–prey mathematical model and assume that the prey population gathers in herds. For the predators, some similar concepts have been discussed in [12]. The prey population that we consider can either live in a $2D$ space, for example the American bisons or other herbivores, or in a $3D$ one, for example in the aquatic or aereal domains as fishes or insects. The model is general enough to accommodate even the case in which the prey can gather in shapes possessing a fractal geometry. In our study R denotes the prey population and F instead the predators. The original idea of this paper is considering a new parameter α, instead of the specific value $1/2$ as used in [4,5]. The relaxed parameter α represents the exponent of the prey population in the interaction term with the predators. As described above, it models the fact that the individuals that stay on the border of the herd are mainly in contact with the predator population. Writing for simplicity R and F instead of $R(t)$ and $F(t)$ respectively, the model reads:

$$\frac{dR}{dt} = r\left(1 - \frac{R}{K}\right)R - aR^\alpha F,$$
$$\frac{dF}{dt} = -mF + aeR^\alpha F. \tag{1}$$

The first equation describes the prey evolution, the first term on the right hand side expresses logistic growth with net reproduction rate r and environment's carrying capacity K; the second term instead models the hunting process they are subject to by predators. The same term, scaled by the conversion coefficient e of prey into predators, appears also in the second equation, in which predators die out with mortality rate m in the absence of their only food resources, namely the prey R. All the parameters are assumed to be nonnegative with $0 < e \le 1$ and $1/2 \le \alpha < 1$. We can remove the singularity of the model (1) in the Jacobian due to the presence of α by defining the new dependent variable $P = R^{1-\alpha}$. We then have

$$\frac{dP}{dt} = (1-\alpha)R^{-\alpha}\frac{dR}{dt} = (1-\alpha)P^{\frac{-\alpha}{1-\alpha}}\frac{dR}{dt}$$