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High-order multioperators-based schemes: Developments and applications

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Abstract

The paper presents the developments of the schemes with multioperators-based approximations for the Euler and the Navier–Stokes equations which orders varying from 9 to 20. The claimed accuracy and resolution of the optimized schemes are illustrated using model equations. The applications to the discontinuous solutions are outlined, the solutions of the Riemann problem with the 20th-order multioperator being presented as an example. The results of high fidelity calculations of flow instabilities phenomena using optimized multioperators-based schemes are presented. They concern with hot jets instabilities manifested by forming vortex rings as well as the upstream propagation of acoustic waves in the case of slightly supersonic underexpanded jets.

Keywords: Multioperators; Optimized schemes; Euler and Navier–Stokes equations; Jets instability; Sound radiation

1. Introduction

The concept of the multioperators proposed in [13,14] and described in [15,16,18,19] can be viewed as an alternative way to increase approximation orders of numerical analysis formulas. As compared with the traditional approach of increasing numbers of *basis functions* defining underlying polynomials, its idea consists of increasing numbers of *basis operators* forming their linear combinations. The basis operators can be readily obtained by fixing parameter's values in various types of one-parameters families of compact approximations. Up to now, applications of the multioperators were aimed at creating very high order schemes capable to perform long-term calculations with high resolution of small details of flows described by the Euler or the Navier–Stokes equations. In particular, the schemes fit neatly into the requirements of DNS, LES and computational aeroacoustics.

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Using multioperators-based schemes, some shear layers instability problems were numerically simulated. The direct numerical simulation of 2D turbulence was reported in [7]. The unsteady behavior of hot axisymmetric subsonic jets generating vortex rings with self-sustained oscillations was investigated in [6]. Very clear pictures of screech waves generated by slightly supersonic 2D underexpanded jets were presented in [21]. The previous experience has shown that the multioperators technique allows one to describe fine flow details using comparatively modest numbers of grid points.

In the present paper, we follow the line of investigations characterized by both developing the multioperators strategy and performing high fidelity calculations with created highly accurate schemes. The outlines of the schemes, their spectral properties and testing examples are presented in Section 2.

Section 3 concerns with the applications of the schemes to the Navier–Stokes numerical simulations of instability phenomena requiring high resolution of flow details during relatively long time calculations. The numerical results show the 2D and 3D screech effects in the case of supersonic underexpanded jets as well as the different vortex rings trains obtained when using axisymmetric and 3D formulations for hot subsonic jets.

2. Multioperators

The general form of a multioperator approximating a linear operator L is

$$L_M(c_1, c_2, \dots, c_M) = \sum_{i=1}^{M} \gamma_i L_m(c_i), \quad \sum_{i=1}^{M} \gamma_i = 1$$

where $L_m(c)$ is a mth order compact approximation to L defined on a uniform mesh ω_h with mesh size h while $(c_1, c_2, \dots, c_M)$ is a set of M different values of c. The γ_i coefficients satisfy the linear system obtained by setting to zero the coefficients for $h^m, h^{m+1}, \dots, h^{m+M-2}$ in terms of the Tailor expansion series for the actions of L_M on sufficiently smooth functions.

Generally, the Taylor expansion series contain both even and odd powers of h. Thus the summation with γ_i coefficients kills $O(h^m), O(h^{m+1}), \dots, O(h^{m+M-2})$ terms in the series providing $O(h^{m+M-1})$ truncation errors.

Multioperators approximating linear operators can be created in the context of various numerical analysis formulas and discretizations. However up to now their main applications like many well-accepted high-order methods (for example, described in [4,8,22,23]) bear relation to fluid dynamics problems.

The high-order optimized schemes in the present study are based on two types of multioperators differing in the ways of using them. The first type is the seventh- and the ninth-order multioperators which basis operators are generated by the fifth-order one-parameter family of compact upwind differencing (CUD) operators (see [18] for details). They require three and five parameters values c_i, $i = 3, 5$ respectively distributed between their minimum and maximum values as zeroth of the Chebyshev polynomials. The Chebyshev distributions are highly desirable since they decrease the condition numbers of the Vandermonde matrices in the systems defining the γ_i coefficients. Three-diagonal inversions are needed to calculate actions of the basis operators on grid functions.

The basis operators form upwind–downwind pairs. It means mathematically that the pairs can be viewed as operators in the Hilbert space of grid functions with summable squares and the inner product defined by the summation over grid points. The operators have the same skew-symmetric components $L_M^{(1)}$ while their self-adjoint components $L_M^{(0)}$ have opposite signs being either positive or negative. So the pairs look as

$$(L_M^{(1)} + L_M^{(0)}, L_M^{(1)} - L_M^{(0)}), \quad L_M^{(0)} > 0.$$

To avoid upwind–downwind switching, the flux splitting was used in the above papers. Considering model equation

$$\frac{\partial u}{\partial t} + \frac{\partial f(u)}{\partial x} = 0, \tag{1}$$

the resulting semi-discretized scheme can be presented in the index-free form as

$$\frac{\partial u}{\partial t} + L_M^{(1)} f(u) + C L_M^{(0)} u = 0, \quad C \geq 0, \tag{2}$$