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A reduced fracture model for two-phase flow with different rock types[☆]

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Abstract

This article is concerned with the numerical discretization of a model for incompressible two-phase flow in a porous medium with fractures. The model is a discrete fracture model in which the fractures are treated as interfaces of dimension 2 in a 3-dimensional simulation, with fluid exchange between the 2-dimensional fracture flow and the 3-dimensional flow in the surrounding rock matrix. The model takes into account the change in the relative permeabilities and in the capillary pressure curves which occurs at the interface between the fracture and the rock matrix. The model allows for barriers which are fractures with low permeability. Mixed finite elements and advective upstream weighting are used to discretize the problem and numerical experiments are shown.

Keywords: Flow in porous media; Discrete fracture model; Two-phase flow; Discontinuous capillary pressure; Mixed finite elements

1. Introduction

The presence of fractures in a porous medium greatly complicates the modeling of flow and transport in a porous medium. Fractures occur on different scales, with different geometries, and may behave either as channels or as barriers for the fluid flow. The fractures thus have a very strong influence on flow and transport, either making flow in certain directions several orders of magnitude more rapid than in other directions or possibly nearly blocking flow in certain directions. There is a need for complex simulation models that resolve the flow dynamics along fractures and the fluid interaction with the porous matrix. The difficulties in the numerical modeling of multiphase flow in fractured media stem from the extremely heterogeneous and anisotropic fracture matrix system and from the nonlinearity due to the relative permeability and capillary pressure. Thus some special method for dealing with these difficulties is required. The so called continuum models take fractures into account through a sort of averaging or homogenizing

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process. Discrete fracture models include the fractures individually in the model. Often in these models it is considered that the amount of flow outside the fractures is negligible and flow in the domain is modeled as flow in a network of fractures which do not communicate with the surrounding medium. More complex discrete fracture models take into account exchange between fractures and the surrounding medium. This last type of model is that of interest for us here, and more precisely we are concerned with reduced fracture models that treat fractures as $(n-1)$-dimensional objects embedded in an n-dimensional medium, n being either 2 or 3, and in which there is communication between the n-dimensional and $(n-1)$-dimensional mediums. A number of articles have been written on numerical models of this type for one-phase flow; see for example [10,3,11,4,37,8,26,40,23,45,21,20] and references therein. For two-phase flow the situation is more complicated due to the change of capillary pressure and relative permeability curves between the fracture and the matrix rock [35,12,33,43,39,30,24,16,42]. In [32] a reduced fracture model which uses the global pressure formulation was introduced which can treat the case of barriers as well as the case of fractures with larger permeability than in the matrix rock, as was done for Darcy flow in [37]. In this paper we present a discretization method of that model using mixed finite elements and show numerical experiments.

After this introduction we recall briefly in Section 2 the global pressure formulation for incompressible two-phase flow. In Section 3 we present a model for incompressible two-phase flow in a fractured domain for which the fracture is modeled as a thin layer in the surrounding matrix, and in Section 4 we show how the reduced model, which treats the fracture as an interface, is obtained. The case of intersecting fractures is considered in Section 5. A multidomain formulation for the reduced model is introduced in Section 6. Numerical methods are discussed in Section 7 and some comments on implementation are made in Section 8. Section 9 shows some numerical experiments.

2. The global pressure formulation for incompressible two-phase flow

We consider incompressible two-phase flow in a porous medium. The governing equations are the equations expressing volume conservation (or equivalently mass conservation since the fluids are assumed to be incompressible) of the two fluid phases and the Darcy law for each phase. For the ℓ phase, $\ell = w$ (wetting phase) or nw (nonwetting phase) the phase conservation equation is

$$\Phi \frac{\partial s_\ell}{\partial t} + \nabla \cdot \mathbf{u}_\ell = q_\ell, \quad \ell \in \{w, nw\}, \tag{1}$$

where s_ℓ is the saturation of the ℓ phase, $\mathbf{u}_\ell$ is its volumetric flow rate, i.e. its Darcy velocity, q_ℓ is the source term given as a function of the phase saturation $(0 \le s_\ell \le 1)$ and Φ is the porosity of the domain. We assume that the volume of all pores is filled by the two phases so that

$$s_w + s_{nw} = 1, \tag{2}$$

and we choose for the main saturation unknown the saturation of the wetting phase $s = s_w$.

The Darcy velocity of the ℓ phase $\mathbf{u}_\ell$ is related to the phase pressure p_ℓ by Darcy's law:

$$\mathbf{u}_\ell = -\mathbf{K}\, k_\ell(s)(\nabla p_\ell - \rho_\ell \mathbf{u}_G), \quad \ell \in \{w, nw\}, \tag{3}$$

where $\mathbf{K}$ denotes the tensor field of absolute permeability, a bounded symmetric uniformly positive definite matrix. ρ_ℓ and k_ℓ are respectively the phase density and the phase mobility, and $\mathbf{u}_G$ denotes the gravity field. The mobilities are positive monotone functions of the saturation s: k_w is increasing with $s = s_w$ and $k_w(0) = 0$, while k_{nw} is decreasing with $s = 1 - s_{nw}$ and $k_{nw}(1) = 0$. Eqs. (1) and (3) taken together constitute the ℓ-phase saturation equation written in mixed form.

The difference between the phase pressures is the capillary pressure π:

$$\pi(s) = p_{nw} - p_w, \tag{4}$$

which is a positive, decreasing function of s. The capillary pressure curve π and the relative permeability curves $k_\ell,\ \ell = w, nw$, depend on the physical properties of the two phases and those of the rock. Among the most frequently used models for describing capillary pressure and relative permeabilities as functions of the saturation are those given by Van Genuchten [17] and those given by Brooks and Corey [31,39]. Further details concerning capillary pressure and relative permeability can be found in [46,18].