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Frequency domain analysis of interacting acoustic–elastodynamic models taking into account optimized iterative coupling of different numerical methods

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ABSTRACT

In this work, interacting acoustic–elastodynamic models are analyzed by means of an optimized iterative coupling algorithm. In this iterative coupling procedure, each acoustic/elastodynamic sub-domain of the model is solved independently, and the variables at the common interfaces of the sub-domains are successively renewed, until convergence is achieved. A relaxation parameter is introduced in order to ensure and/or speed up the convergence of the iterative analysis, and an expression to compute optimal values for the relaxation parameter is presented. Several numerical methods are considered to discretize the acoustic and elastodynamic sub-domains of the coupled model, and the performance of these different methodologies, in the coupled analysis, is discussed. In this context, the boundary element method and the method of fundamental solutions are applied to model the acoustic sub-domains, whereas the finite element method, the collocation method and the meshless local Petrov–Galerkin method are applied to model the elastodynamic sub-domains. Independent discretizations of the acoustic/elastodynamic sub-domains are allowed, being no matching nodes required along the common interfaces. At the end of the paper, numerical examples are presented, illustrating the performance and potentialities of the adopted procedures.

1. Introduction

In engineering analysis there is not a single numerical method that can handle properly all problems and it did not take long until some researchers started to combine different methodologies, in order to profit from their advantages, trying to evade their disadvantages.

Considering hyperbolic applications, the first works considering the combination of different numerical procedures were concentrated in the establishment of a coupled system of equations, taking into account distinct pre-selected discretization methods, as reported by manuscripts dealing with time [1–3] and frequency [4–6] domain analyses. Later on, iterative coupling algorithms have been proposed, considering once again time [7–9] and frequency [10–12] domain approaches. In the iterative coupling approach, each sub-domain of the global model is analyzed independently (as an uncoupled model) and a successive renewal of the variables at the common interfaces is performed, until convergence is achieved. These iterative methodologies exhibit several advantages when compared to standard coupling schemes, as for instance: (i) different sub-domains can be analysed separately, leading to smaller and better-conditioned systems of equations (different solvers, suitable for each sub-domain, may be employed); (ii) only interface routines are required when one wishes to use existing codes to build coupling algorithms (thus, coupled systems may be solved by separate programme modules, taking full advantage of specialized features and disciplinary expertise); (iii) matching nodes at common interfaces are not required, greatly improving the flexibility and versatility of the coupled analyses, especially when different discretization methods are considered; (iv) more efficient analyses can be obtained, once the global model can be reduced to several sub-domains with reduced size matrices; etc.

In the present work, an optimised frequency domain iterative coupling algorithm is presented to analyze interacting acoustic–elastodynamic models, which are discretized by several different numerical methods. As it has been reported [10,11], frequency domain analyses usually give rise to ill-posed problems and, in these cases, the convergence of simple iterative coupling algorithms can either be too slow or unachievable. In order to deal with this ill-posed problem and ensure convergence of the

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iterative coupling algorithm, an optimal iterative procedure is adopted here, with optimal relaxation parameters being computed at each iterative step. As it is described along the paper, the introduction of these optimal relaxation parameters allows the iterative coupling technique to be very effective in the frequency domain, ensuring convergence at a low number of iterative steps.

Several numerical methods are adopted here in order to discretize the acoustic and elastodynamic sub-domains. This is especially important since the numerical analysis of acoustic–elastodynamic coupled systems is a complex task, requiring proper treatment of sub-domains in which different physical phenomena are involved, as well as suitable numerical modelling of wave propagation across arbitrary shaped interfaces. In this context, the boundary element method [13,14] and the method of fundamental solutions [15,16] are here applied to discretize the acoustic sub-domains, whereas the finite element method [17,18] and meshless methods based on collocation [19,20] and on local Petrov–Galerkin discretizations [20,21] are applied to model the elastodynamic sub-domains. Making use of these different numerical approaches within the present paper, it becomes possible, on one hand, to show the independence of the proposed iterative coupling strategy with respect to the adopted methods, and, on the other, to evidence the relative advantages and disadvantages of each method for analysing some particular configurations. Thus, a comparison between the effectiveness of these different methodologies can be carried out taking into account acoustic–elastodynamic coupled analyses. As it is well known, the boundary element method and the method of fundamental solutions are boundary discretization techniques, and they are very appropriate to model infinite and semi-inifinite media, which is usually the case when acoustic fluids are considered. On the other hand, the finite element method and the meshless methods discussed here are domain discretization techniques, being more appropriate to analyze media with complex physical behaviour (heterogeneities, anisotropy etc.), which is usually the case considering dynamic solids.

The paper is organized as follows: first, the governing equations of the physical problem are presented; then, the focused discretization methods are briefly discussed. In the sequence, the iterative coupling technique is described, including the mathematical derivation of the optimisation methodology. At the end of the paper, numerical applications are presented, illustrating the accuracy, performance and potentialities of the proposed procedures.

2. Governing equations

In this section, acoustic and elastodynamic governing equations are briefly presented, as well as their coupling conditions. Integral formulations, which are directly employed by the different numerical methods focused here, are presented at the end of the section.

2.1. Acoustic sub-domains

The acoustic scalar wave equation is given by

$$p(X,\omega)_{,ii}+\gamma^2 p(X,\omega)+s(X,\omega)=0 \qquad (1)$$

where $p(X,\omega)$ and $s(X,\omega)$ stand for hydrodynamic pressure distribution and body source terms, respectively. Indicial notation is adopted and inferior commas indicate partial space derivatives ($p_{,i}=\partial p/\partial x_i$). $\gamma=\sqrt{\omega^2/c^2-\mathrm{i}\omega\nu/\kappa}$ stands for the complex wave number, where $c=\sqrt{\kappa/\rho}$ is the wave propagation velocity and ν, ρ and κ stand for the viscous damping coefficient, the mass density and the bulk modulus of the medium, respectively. The boundary conditions of the problem are given by

$$p(X,\omega)=\overline{p}(X,\omega)\ \text{for}\ X\in\Gamma_1 \qquad (2a)$$

$$q(X,\omega)=p_{,j}(X,\omega)n_j(X)=\overline{q}(X,\omega)\ \text{for}\ X\in\Gamma_2 \qquad (2b)$$

where the prescribed values are indicated by over bars and $q(X,\omega)$ represents the flux along the boundary whose unit outward normal vector components are represented by $n_j(X)$. The boundary of the model is denoted by $\Gamma(\Gamma_1\cup\Gamma_2=\Gamma$ and $\Gamma_1\cap\Gamma_2=0$, where Γ_1 stands for the essential or Dirichlet boundary and Γ_2 stands for the natural or Neumann boundary) and the domain by Ω.

2.2. Elastodynamic sub-domains

The frequency domain elastodynamic equations are given by

$$\sigma_{ij}(X,\omega)_{,j}+(\rho\omega^2-\mathrm{i}\omega\rho\nu)u_i(X,\omega)+\rho b_i(X,\omega)=0 \qquad (3a)$$

$$\sigma_{ij}(X,\omega)=\lambda\delta_{ij}\varepsilon_{kk}(X,\omega)+2\mu\varepsilon_{ij}(X,\omega) \qquad (3b)$$

$$\varepsilon_{ij}(X,\omega)=1/2(u_i(X,\omega)_{,j}+u_j(X,\omega)_{,i}) \qquad (3c)$$

where $u_i(X,\omega)$ and $b_i(X,\omega)$ stand for the displacement and the body force distribution components, respectively. The notation for space derivatives employed in Eq. (1) is once again adopted. ρ is the mass density, λ and μ are the Lamé's constants and ν stands for viscous damping related parameters. $\sigma_{ij}(X,\omega)$ and $\varepsilon_{ij}(X,\omega)$ are, stress and strain tensor components, respectively and δ_{ij} is the Kronecker delta ($\delta_{ij}=1$, for $i=j$ and $\delta_{ij}=0$, for $i\neq j$). Eq. (3a) is the momentum equilibrium equation; Eq. (3b) represents the constitutive law of the linear elastic model and Eq. (3c) stands for kinematical relations. The boundary conditions of the elastodynamic problem are given by

$$u_i(X,\omega)=\overline{u}_i(X,\omega)\ \text{for}\ X\in\Gamma_1 \qquad (4a)$$

$$\tau_i(X,\omega)=\sigma_{ij}(X,\omega)n_j(X)=\overline{\tau}_i(X,\omega)\ \text{for}\ X\in\Gamma_2 \qquad (4b)$$

where $\tau_i(X,\omega)$ denotes the traction vector along the boundary.

2.3. Acoustic–elastodynamic interacting interfaces

On the acoustic–elastodynamic interface boundaries, the dynamic sub-domain normal (normal to the interface) displacements ($u_n(X,\omega)$) are related to the acoustic sub-domain fluxes ($q(X,\omega)$), and the acoustic sub-domain hydrodynamic pressures ($p(X,\omega)$) are related to the dynamic sub-domain normal tractions ($\tau_n(X,\omega)$). These relations are expressed by the following equations:

$$u_n(X,\omega)+1/(\rho\omega^2)q(X,\omega)=0 \qquad (5a)$$

$$\tau_n(X,\omega)+p(X,\omega)=0 \qquad (5b)$$

where ρ stands for the mass density of the interacting acoustic sub-domain medium.

2.4. Integral equations

Taking into account a generic matricial notation, the strong, weak and inverse integral forms of the acoustic and elastodynamic governing equations correspondingly can be written as follows:

$$\int_\Omega \mathbf{v}^T(\mathbf{L}_d^T\mathbf{D}\mathbf{L}_d\mathbf{y})d\Omega+\int_\Omega \mathbf{v}^T\varsigma\mathbf{y}d\Omega+\int_\Omega \mathbf{v}^T\boldsymbol{\beta}d\Omega=0 \qquad (6a)$$

$$\int_\Omega(\mathbf{v}^T\mathbf{L}_d^T)\mathbf{D}\mathbf{L}_d\mathbf{y}d\Omega-\int_\Gamma \mathbf{v}^T(\mathbf{L}_n^T\mathbf{D}\mathbf{L}_d\mathbf{y})d\Gamma-\int_\Omega \mathbf{v}^T\varsigma\mathbf{y}d\Omega-\int_\Omega \mathbf{v}^T\boldsymbol{\beta}d\Omega=0 \qquad (6b)$$