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A stabilized meshless method for time-dependent convection-dominated flow problems

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Abstract

Meshless methods for convection-dominated flow problems have the potential to reduce the computational effort required for a given order of solution accuracy compared to mesh-based methods. The state of the art in this field is more advanced for elliptic partial differential equations than for time-dependent convection–diffusion problems. In this paper, we introduce a new meshless method that it based on combining the modified method of characteristics with the radial basis functions during the solution reconstruction. The method belongs to a class of fractional time-stepping schemes in which a predictor stage is used for the discretization of convection terms and a corrector stage is used for the treatment of diffusion terms. Special attention is given to the application of this method to solve convection-dominated flow problems in two-dimensional domains. Numerical results are shown for several test examples including the incompressible Navier–Stokes equations and the computed results support our expectations for a stable and highly accurate meshless method.

Keywords: Meshless method; Radial basis functions; Modified method of characteristics; Convection-dominated flows; Incompressible Navier–Stokes equations

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1. Introduction

Convection–diffusion equations have been widely used to model many applications in physical and engineering areas such as weather prediction, ocean circulation, petroleum reservoir etc. The common characteristic in these applications is convective terms are distinctly more important than the diffusive terms; particularly when the Peclet number reaches high values. It is also well-established that these convective terms are a source of computational difficulties and oscillations. On the other hand steep fronts, shocks and boundary layers are among the difficulties that most of numerical methods fail to resolve accurately, see for instance [20]. It is well known that the mesh-based methods use fixed grids and incorporate some upstream weighting in their formulations to stabilize the schemes. These mesh-based methods include the Petrov–Galerkin methods, the streamline diffusion methods and also many other methods such as the high resolution methods from computational fluid dynamics, in particular, the Godunov methods and the essentially non-oscillatory methods, see [8,28] among others. The main shortcoming of these methods lies on the fact that the accuracy of these methods is affected by the quality of the meshes, stabilization techniques and solution of Riemann problems, which hinders their applications to solving real problems with irregular domains and complex Riemann problems.

Recently, significant developments in meshless methods for solving linear as well as nonlinear partial differential equations have been achieved. For instance, the meshless local Petrov–Galerkin and local boundary integral equations methods were investigated in [1,2]. These methods basically transformed the original problem into a local weak formulation and the shape functions were constructed from using the moving least-squares approximation to interpolate the solution variables. The meshless radial basis functions (RBF) have been subject to several studies and their applications to solve partial differential equations have also been covered in the literature. The RBF approximations, particularly the multiquadric basis functions, were first devised for scattered geographical data interpolation in [31,19]. In the interpolation framework, a review on the application of RBF methods for scattered data can be found in [15]. Theoretical results for RBF methods have also been presented in [5,21] among others. These results include solvability, convergence and stability of the RBF interpolation in a general concept. Application of the RBF methods to steady and time-dependent partial differential equations has also been investigated, see for example [13,16]. Recent local formulations of RBF methods have been achieved during this last years [3,26,25,30,24] and some of these local RBF methods have been used to solve hyperbolic systems of conservation laws such as Euler system of gas dynamics and shallow water equations in [27,32,17,7,23]. However, for practical applications in hyperbolic systems, these methods may become computationally demanding due to Riemann solvers in their implementations.

In the current study we propose a stabilized meshless method for convection-dominated flow problems which is simple, accurate and Riemann solver free. This is achieved by combining the Modified Method of Characteristics (MMC) with a class of local radial basis functions. Because of the hyperbolic nature of advective transport in the considered equations, the MMC has been successfully applied to solve convection-dominated flow problems. The MMC carries out the temporal discretization by following the movement of particles along the characteristic curves of the governing equations, see for example [9,29]. Because the solution of advective part is much smoother along the characteristics than they are in the time direction, MMC eliminates the stability restrictions on the Courant number and generates accurate solutions even if large time steps are used in the computations. The idea of developing stable meshless methods to integrate partial differential equations has a long tradition for elliptic class of these equation. This field of research is very active for elliptic equations, where a vast number of numerical schemes have been designed based on global as well local RBF techniques. All of these meshless methods are easy to formulate and to implement. However, their direct application to transient partial differential equations of hyperbolic type results in instabilities, presence of nonphysical oscillations and poor resolution of shocks. The main focus of our work is the development of a truly meshless RBF method to numerically solve the convection-dominated diffusion problems. The key idea in the current approach is a predictor–corrector solver for which the MMC is used in the predictor stage whereas the corrector stage uses the RBF method. The results using the proposed meshless method are presented for three test problems. To the best of our knowledge, solving convection-dominated flow problems using these numerical tools is reported for the first time.

This paper is organized as follows. In Section 2 we present the stabilized meshless method for time-dependent convection–diffusion problems. This section includes both the first fractional step used in the predictor stage to resolve the convection terms and the second fractional step used in the corrector stage for the diffusion terms. Implementation of modified method of characteristics is also covered in this section. Numerical results and examples are presented in