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Study on harbor resonance and focusing by using the null-field BIEM

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ABSTRACT

In this paper, the resonance of a circular harbor is studied by using the semi-analytical approach. The method is based on the null-field boundary integral equation method in conjunction with degenerate kernels and the Fourier series. The problem is decomposed into two regions by employing the concept of taking free body. One is a circular harbor, and the other is a problem of half-open sea with a coastline subject to the impermeable (Neumann) boundary condition. It is interesting to find that the SH wave impinging on the hill can be formulated by the same mathematical model. After finding the analogy between the harbor resonance and hill scattering, focusing of the water wave inside the harbor as well as focusing in the hill scattering are also examined. Finally, two numerical examples, circular harbor problems of 60° and 180° opening entrance, are both used to verify the validity of the present formulation.

1. Introduction

Hydrodynamic design of a harbor or a marina is mainly aimed to obtain a sheltered area to berth safely and comfortably for boats. For this purpose, it is unavoidably related to the period and the amplitude of the oscillations of the free surface. In some situations, the wave amplitude inside the harbor at a particular location may be much higher than the amplitude of the incident wave for a certain wavenumber, whereas for another case of wave periods significant attenuation may appear at the same place. This phenomenon of harbor oscillation has generally been thought to be caused by waves from the open-sea incident upon the harbor entrance, although other possible excitations may be from local winds, earthquakes, local atmospheric pressure anomalies, etc.

Mathematically speaking, harbor resonance is different from the eigenproblem since the Dirichlet to Neumann (DtN) conditions in the harbor entrance are not known. Harbor oscillation may be due to a resonance of physical behavior induced by the incident wave from the entrance. Several investigators have paid attention to this issue. Both theoretical and experimental studies have been carried out by many researchers. For example, McNown [1] investigated the forced oscillation in the harbor of a circular boundary with a narrow opening. Kravtchnenko and McNown [2] analyzed the effect of forced oscillation in a rectangular harbor. Miles and Munk [3] were the first to theoretically study the problem of a rectangular harbor with a real opening at the entrance, thus obtaining bounded amplitudes of waves at resonance. Ippen and Goda [4] have studied the effect of the entrance location of a harbor. They also studied the problem of a rectangular harbor connected to the open-sea. Lee [5,6] investigated the response characteristics of a circular harbor with 10° and 60° opening entrances, to find the harbor resonance. Chen [7] used the composite BEM to solve the circular harbor problem with 10° and 60° opening entrances. The crest of a standing wave occurred at the entrance. In addition, Lee [5] performed experiments which were in good agreement with the data by using the linearized water wave theory. Mei and Ünlüata [8] developed a method to solve the problem of wave-induced oscillation in a harbor with two coupled basins with a constant water depth. Transient response of harbors to long waves under resonance conditions was also investigated by Bellotti [9]. Besides, the issue of corner singularity has been analytically studied by Linton [10].

Following the successful experience of the null-field boundary integral equation method (BIEM) for solving the boundary value problems (BVPs) containing circular boundaries, five advantages such as mesh-free generation, well-posed model, principal-value free, elimination of boundary-layer effect and exponential convergence have been achieved. Applications to various fields, e.g., torsion bar [11], antiplane shear [12], piezoelectricity [13], bending of cantilever beam [14], acoustics [15], membrane vibration [16], SH-wave [17], Stokes' flow [18], and plate vibration problems [19] have been done. Application of the null-field BIEM to water-wave problem [20–23] has also been done. Regarding a circular harbor, we intend to revisit this problem by using the null-field BIEM as shown in Fig. 1(a).

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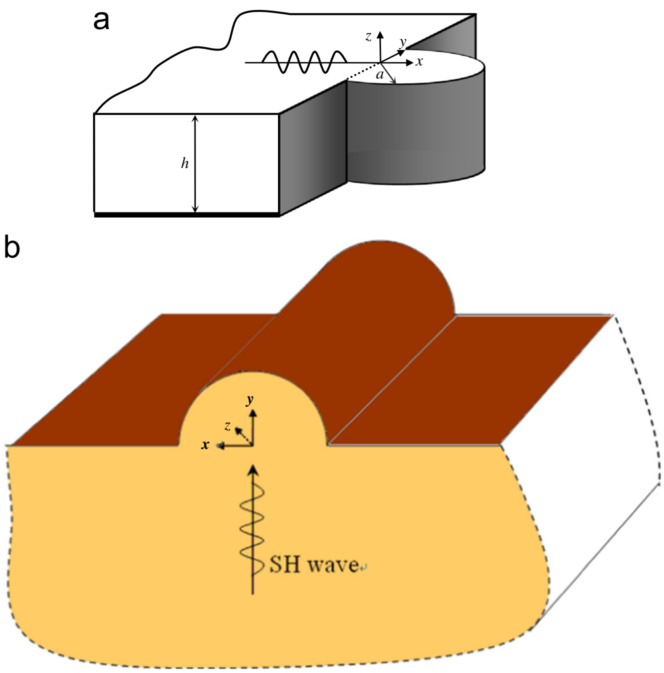

Fig. 1. Problem sketch of two engineering problems. (a) a circular or semi-circular harbor, and (b) a semi-circular hill.

The present work is to study the oscillation in the harbor with a circular arc boundary by using the null-field BIEM for the Helmholtz equation. A circular harbor with 60° entrance will be demonstrated to see the validity of the present formulation. Besides, the semi-circular harbor problem is also solved. But no data can be compared within the harbor literature to the authors' best knowledge. Nevertheless, we find that the hill scattering of SH waves can be formulated into the same mathematical model. The main difference between the harbor resonance and hill scattering in physics is the medium. One is water and the other is soil, as shown in Fig. 1(a) and (b), respectively. A list of similarity and difference between harbor resonance and hill scattering is given in Table 1. Mathematically speaking, the two problems can be described by using the same governing equation (Helmholtz) and boundary conditions (Neumann). The focusing effect in optics, acoustics and electromagnetics as well as for elastic waves has been noticed [24], for example, Northridge earthquake damage from the geologic focusing of seismic waves [25]. Tsaur and Chang [24] employed the wave function expansion approach to find the focusing behavior for the shallow circular arc hill in both time and frequency domains. The maximum response may occur beneath the hill boundary which may cause failure for underground structures. It is important that the maximum amplitude does not happen on the ground surface, but appears under the ground. We may wonder whether the focusing effect may happen in the harbor resonance or not. From the viewpoint of berthing safely for boats, it is not trivial to study the focusing phenomenon in harbor resonance. Therefore, the developed program is utilized to solve the two different physical problems together and to check the accuracy of both results. Also, focusing is examined in this paper.

2. Problem statement

A circular harbor along the coastline is considered, and assumes the incident wave coming from infinity with the constant water depth of h as shown in Fig. 1(a). The governing equation of the water wave problem is the Laplace equation as follows:

$$\nabla^2 \Phi(\mathbf{x}, z; t) = 0, \quad (\mathbf{x}, z) \in D, \tag{1}$$

where ∇^2 and D are the Laplacian operator and the domain of interest, respectively, $\Phi(\mathbf{x}, z; t)$ is the velocity potential and $\mathbf{x} = (x, \ y)$ is the plane position vector in terms of the Cartensian coordinates. According to the method of separation variables, the velocity potential can be written as

$$\Phi(\mathbf{x}, z; t) = u(\mathbf{x}) f(z) e^{-i\omega t}, \tag{2}$$

where ω is the angular frequency. The boundary condition at the bottom and the linear kinematic and dynamic boundary conditions on the free surface are shown below

$$\frac{\partial \Phi(\mathbf{x}, z; t)}{\partial z} = 0, \ z = -h, \tag{3}$$

$$\frac{\partial \Phi(\mathbf{x}, z; t)}{\partial z} = \frac{\partial \eta(\mathbf{x}; t)}{\partial t}, z = 0 \tag{4}$$

$$\frac{\partial \Phi(\mathbf{x}, z; t)}{\partial t} + g\eta(\mathbf{x}; t) = 0, z = 0 \tag{5}$$

respectively, where η is the free-surface elevation which can be determined by

$$\eta(\mathbf{x}; t) = Au(\mathbf{x}) e^{-i\omega t} = \frac{i\omega}{g} \Phi(\mathbf{x}, 0; t), \tag{6}$$

and A is the amplitude of the incident wave and g is the acceleration due to gravity. In order to satisfy Eqs. (3) and (6), the function $f(z)$ can be found as

$$f(z) = \frac{-igA}{\omega} \frac{\cosh(k(z+h))}{\cosh(kh)}. \tag{7}$$

The dispersion relationship is obtained by satisfying Eq. (4)

$$k \tanh(kh) = \frac{\omega^2}{g} \tag{8}$$

Substituting Eq. (2) into Eq. (1), we have

$$(\nabla^2 + k^2) u(\mathbf{x}) = 0, \quad \mathbf{x} \in D \tag{9}$$

It is noted that the field $u(\mathbf{x})$ in the region I and II is expressed by $u^I(\mathbf{x}) + u^R(\mathbf{x}) + u^O(\mathbf{x})$ and $u^H(\mathbf{x})$, respectively. The fields $u^I(\mathbf{x})$ and $u^R(\mathbf{x})$ mean incident and reflected waves, respectively. The wall of a circular harbor and coastline are subject to the Neumann boundary condition as shown below

$$\frac{\partial u^H(\mathbf{x})}{\partial n_{\mathbf{x}}} = 0, \quad \mathbf{x} \in B^H, \tag{10}$$

$$\frac{\partial u^O(\mathbf{x})}{\partial x} = 0, \quad \mathbf{x} \in B^C, \tag{11}$$

where $n_{\mathbf{x}}$ denotes for the unit normal vector along the circular harbor with respect to its local polar coordinate system, B^H and B^C denote the wall of a circular harbor and coastline, respectively as shown in Fig. 2(a). Besides, the field $u^O(\mathbf{x})$ also satisfies the radiation condition at infinity as shown below

$$\lim_{|\mathbf{x}| \to \infty} |\mathbf{x}|^{1/2} \left(\frac{\partial}{\partial |\mathbf{x}|} - ik \right) u^O(\mathbf{x}) = 0. \tag{12}$$

The incident wave of θ_{inc} angle is given as follows:

$$u^I(\mathbf{x}) = e^{ik(x\cos\theta_{inc} + y\sin\theta_{inc})} \Rightarrow u^I(\rho, \phi) = e^{ik\rho(\cos(\phi)\cos(\theta_{inc}) + \sin(\phi)\sin(\theta_{inc}))}, \tag{13}$$

where $(\rho, \ \phi)$ are polar coordinates of the point $\mathbf{x}$.