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The dynamic behaviors of one-predator two-prey system with mutual interference and impulsive control

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Abstract

Taking into account chemical control, biological control for pest management at different fixed moments, and mutual interference of the predator. A one-predator two-prey system with impulsive effects and mutual interference is established in this paper. By using techniques of impulsive perturbations, Floquet theory and comparison theorem, we investigate the existence and globally asymptotic stability of prey-eradication periodic solution. We also derive some sufficient conditions for the permanence of the system by using comparison methods involving multiple Lyapunov functions. Our results improve some obtained results. Then numerical simulations are given to show the complex behaviors of this system. Finally, we analyze the biological meanings of these results and give some suggestions for feasible control strategies.

Keywords: Mutual interference; Impulsive control; Extinction; Permanence

1. Introduction and model formulation

In real world, models of three or more species, such as three species predator–prey systems and food-chain systems, are popular and have extremely rich dynamics [16,17]. For predator–prey model, in description of the relationship between predator and prey, a crucial element is the classic definition of a predator's functional response. Recently, the dynamic behaviors of predator–prey system with different kinds of functional responses have been extensively investigated [2,3,8,11]. For example, the authors [8] gave the following non-linear functional response:

$$f_i(x_1, x_2) = \frac{c_i x_i}{d_1 + d_2 x_1 + d_3 x_2}, \quad i = 1, 2,$$

where c_i is the rate of a predator searching for the prey x_i, d_1 is a positive constant, $d_2 = h_1 c_1, d_3 = h_2 c_2, h_i$ represents the expected handling time spent with the prey x_i to predator x_3. Obviously, if d_2 or d_3 tends to zero, it will be Holling type-II functional response [4].

It is well known that insects have a very important influence on human life. Many kinds of insects are beneficial to agricultural production, but a few are harmful to agricultural and economic development when they reach a certain

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amount. Hence it is necessary to control insects in a suitable mount. Chemical control and biological control are very important methods for agricultural pest control. By spraying pesticides, chemical control strategy is used broadly because it can effectively kill pests and reduce the economic losses, but it often affects the environments. For less pollution to the environment, by stocking or releasing natural enemies, biological control appears, but the effects are not very great. In order to combine different approaches to control pests at the same time, integrated pest management is given to maximize control efficiency and reduce pollution. During the last two decades, controlling insects and other arthropods have become an increasingly complex project [12,15]. For predator–prey system, pest control strategy has been an important topic for many scholars [5,10,18]. Pei [13] studied the dynamic behaviors of the following one-predator two-prey model with integrated impulsive controls:

$$
\begin{cases}
\left.\begin{aligned}
x_1'(t) &= r_1x_1(t)\left(1-\frac{x_1(t)}{k_1}\right)-\frac{\alpha_1x_1(t)x_3(t)}{a_1+b_1x_1(t)+b_2x_2(t)},\\
x_2'(t) &= r_2x_2(t)\left(1-\frac{x_2(t)}{k_2}\right)-\frac{\alpha_2x_2(t)x_3(t)}{a_1+b_1x_1(t)+b_2x_2(t)},\\
x_3'(t) &= \left(-d+\frac{m_1\alpha_1x_1(t)+m_2\alpha_2x_2(t)}{a_1+b_1x_1(t)+b_2x_2(t)}\right)x_3(t),
\end{aligned}\right\}
\begin{array}{l} t\neq k\tau,\\ t\neq (k+l-1)\tau, \quad k\in N,\end{array}\\
\left.\begin{aligned}
x_1(t^+) &= (1-\mu_1)x_1(t),\\
x_2(t^+) &= (1-\mu_2)x_2(t),\\
x_3(t^+) &= x_3(t),
\end{aligned}\right\} t=(k+l-1)\tau, \quad k\in N,\\
\left.\begin{aligned}
x_1(t^+) &= x_1(t),\\
x_2(t^+) &= x_2(t),\\
x_3(t^+) &= x_3(t)+p,
\end{aligned}\right\} t=n\tau, \quad k\in N,
\end{cases}
$$

where $x_1(t), x_2(t), x_3(t)$ are the densities of the two preys and a predator at time t, respectively.

However, in the actual ecosystem, few researchers consider the mutual interference between predators, but mutual interference between predators always exists. In 1971, by researching the hosts capturing behavior for parasites, Hassell found this phenomenon that, if parasites or hosts met, they will deviate from each other, which interferes the effects of hosts capturing. If the size of the parasite became larger, then the mutual interference would be stronger. Hence he introduced the mutual interference of predators [6]. Taking into account the effect from mutual interference between predators, the dynamic behaviors are more complex. For example, He [7] studied the mutual interference of the predator and obtained much different dynamics with those models without mutual interference. Zhang [19] also studied the mutual interference of the predator in depth. Hence, it is necessary to take into account the mutual interference from the predator. The main purpose of this paper is to construct a one-predator two-prey model with mutual interference and integrated control methods and to investigate the dynamical behaviors. The model is portrayed by the following impulsive differential equations:

$$
\begin{cases}
\left.\begin{aligned}
x_1'(t) &= x_1(t)(a_1-b_1x_1(t))-\frac{c_1x_3(t)^m x_1(t)}{d_1+d_2x_1(t)+d_3x_2(t)},\\
x_2'(t) &= x_2(t)(a_2-b_2x_2(t))-\frac{c_2x_3(t)^m x_2(t)}{d_1+d_2x_1(t)+d_3x_2(t)},\\
x_3'(t) &= -a_3x_3(t)+\left(\frac{k_1c_1x_1(t)+k_2c_2x_2(t)}{d_1+d_2x_1(t)+d_3x_2(t)}\right)x_3(t)^m,
\end{aligned}\right\}
\begin{array}{l} t\neq nT,\\ t\neq (n+l-1)T,\end{array}\\
\left.\begin{aligned}
x_1(t^+) &= (1-\mu_1)x_1(t),\\
x_2(t^+) &= (1-\mu_2)x_2(t),\\
x_3(t^+) &= (1-\mu_3)x_3(t),
\end{aligned}\right\} t=(n+l-1)T,\\
\left.\begin{aligned}
x_1(t^+) &= x_1(t),\\
x_2(t^+) &= x_2(t),\\
x_3(t^+) &= x_3(t)+p,
\end{aligned}\right\} t=nT,
\end{cases}
\tag{1.1}
$$

where $x_1(t), x_2(t), x_3(t)$ are densities of two preys and one predator at time t, respectively. a_i $(i=1,2)$ is intrinsic increasing rate, a_3 is the death rate of predator. m represents the mutual interference of the predator, $0<m\leq 1$. k_i $(i=1,2)$ is transformation rate for the predator to prey, b_i $(i=1,2)$ is death rate of prey. $0<\mu_i<1$ $(i=1,2,3)$