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Nonlinear Cournot and Bertrand-type dynamic triopoly with differentiated products and heterogeneous expectations

J. Andaluz[a,*], A.A. Elsadany[b], G. Jarne[a]

[a] *Department of Economic Analysis, University of Zaragoza, Zaragoza, Spain*
[b] *Department of Basic Science, Faculty of Computers and Informatics, Suez Canal University, Ismailia 41522, Egypt*

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Abstract

In a differentiated triopoly model with heterogeneous firms, the local stability of the Nash equilibrium under both quantity and price competition is analyzed. We find that the presence of a firm following a gradient rule based on marginal profits, and a player with adaptive expectations, determines the local stability of the Nash equilibrium, regardless the competition type, while the effects of the degree of product differentiation on the stability depend on the nature of products. Moreover, the Nash equilibrium is more stable under quantity competition than under price competition.

Keywords: Bertrand competition; Cournot competition; Heterogeneous expectations; Stability; Bifurcation

1. Introduction

Two classical models in the theory of oligopoly are those of Cournot [13] and Bertrand [7]. In Cournot's model the firms choose to compete on output quantity, and in Bertrand's model they choose to compete on price. Both models can be interpreted as static games where decisions are made simultaneously and where each firm maximizes its own profit, in a context of perfect and complete information.[1]

However, there is a growing interest in analyzing Cournot and Bertrand competition in a dynamic setting. Assuming the discrete time scale, the properties of the resulting dynamic process are given by the way that the firms adjust their quantity or price levels that, in turn, depends on the formation of their expectations.

The naïve, or Cournot, expectations assume that firms use the latest available information. Thus, each firm expects its rivals to offer the same quantity or price in the current period as they did in the previous period and there is no retaliation. In this setting, it is concluded that for the duopoly case, the equilibrium is globally stable as it is deduced in the static context. However, under this expectation rule, in a seminal paper, Theocharis [24] showed that in a Cournot oligopoly with linear demand function and constant marginal costs, an increasing in the number of competitors plays

* Correspondence to: Department of Economic Analysis, University of Zaragoza, Gran Via, 2, 50005 Zaragoza, Spain. Fax: +34 976761966.
E-mail address: jandaluz@unizar.es (J. Andaluz).

[1] Otherwise, the Cournot and Bertrand models can be interpreted as conjectural variation models (see [10]). In Cournot's original model, each firm's conjecture is that the other firms are satisfied to continue selling their current quantity of output. However, from a purely game-theory perspective, the conjectural variations approach is theoretically unsatisfactory (see [25]).

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a destabilizing role. Particularly, if the number of firms is higher than three, the Cournot–Nash equilibrium becomes unstable. This paradoxical result has been generalized by varying different assumptions, for example, by considering different shapes of the demand function (see [4,21]), of the cost function [19], or both [28].

The burgeoning interest in nonlinear dynamic oligopolies has renewed the use of decisional mechanism. In this line, more realistic expectation rules than naïve expectations have been proposed. It is the case of gradient rule based on marginal profits (see [8,23,5]), and the adaptive expectations principle [9].

In the literature, most papers focus on games with homogeneous players, that is, firms that adopt the same expectation rule. However, it may be more realistic to assume that firms have heterogeneous expectations. Belonging to this group, we can cite [1,2,26], who analyzed a duopoly model with homogeneous product, and assuming heterogeneous expectations. More recently, Fanti and Gori [16] analyze the dynamics of a horizontally[2] differentiated duopoly under Cournot competition, with heterogeneous players.

The analysis of oligopolies with more than two firms has been less addressed in the literature. In this line, [20,3,11] are examples of homogeneous triopolies. The research on dynamics in games with more than two heterogeneous players is still poor. As exceptions, we can cite Elsadany et al. [15], who consider an oligopoly game with four heterogeneous firms producing perfect substitute goods, and show that the stability of the Cournot–Nash equilibrium depends on the speed of adjustment of the gradient firm, and the player with adaptive expectations has a stabilizing effect on the game. On the other hand, in an oligopoly model with isoelastic inverse demand function and constant marginal costs, Tramontana et al. [27] show that the stability region on the parametric space may enlarge by increasing the number of heterogeneous competitors.

The case of three heterogeneous firms producing differentiated goods is considered by Andaluz and Jarne [6]. These authors study a linear dynamic system of a differentiated triopoly under quantity and price competition, and assuming that two firms follow a gradient rule based on marginal profits and a firm revises its beliefs according to naïve expectations. It is showed that in the presence of both complement and substitute goods, the stability of the Nash equilibrium increases when goods tend to be independents. Moreover, the Nash equilibrium is more stable under quantity competition than under price competition.

The present article constitutes an extension of the model proposed by [6]. Firstly, we analyze the local stability of the Nash equilibrium, under both quantity and price competition, but assuming a nonlinear dynamic system of a differentiated triopoly. Secondly, we introduce more heterogeneity among players. Thus, we assume gradient rule, naïve expectations, and adaptive expectations hypothesis for the three firms competing in the market, respectively.

We conclude that there is a critical value of the adjustment speed of the gradient player, for which the Nash equilibrium loses its stability. This threshold is higher under Cournot competition than under Bertrand competition, and therefore, the Cournot–Nash equilibrium is more stable than the Bertrand–Nash equilibrium, regardless of the nature of products.

On the other hand, both under Bertrand and Cournot competition, it is deduced that, in the presence of complement goods, a lower degree of product differentiation (goods tend to be independent) stabilizes the Nash equilibrium. By contrast, assuming substitutes goods, there exists a level of product differentiation that ensures the maximal stability of the Nash equilibrium.

The remainder of the paper is organized as follows. Section 2 presents the model that is developed for this study. Section 3 develops the dynamics under Cournot competition. The dynamics under price competition is analyzed in Section 4. Section 5 closes the paper with the main conclusions.

2. The model

We extend the model formulated by [22] for the triopoly case. Specifically, we consider an economy with a monopolistic sector of three firms, each producing a differentiated good, and a competitive numeraire sector. There is a continuum of identical consumers with a utility function separable and linear in the numeraire good.

Denoting as q_i the quantity of good i, and p_i its price, the representative consumer maximizes the following utility function with respect to the quantities:

[2] Horizontal product differentiation has been developed through the non-address approach, dating back to [10,12], and the address models, dating back to [18].