Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 132 (2017) 100–119
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Risk measurement of a guaranteed annuity option under a stochastic modelling framework

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Received 20 February 2015; received in revised form 2 December 2015; accepted 6 July 2016
Available online 16 July 2016

Abstract

We address the problem of setting capital reserves for a guaranteed annuity option (GAO). The modelling framework for the loss function of GAO is developed. A one-decrement actuarial model is considered in which death is the only decrement, and the interest and mortality risk factors follow correlated affine structures. Risk measures are determined using moment-based density method and benchmarked with the Monte-Carlo simulation. Bootstrap technique is utilised to assess the variability of risk measure estimates. We establish the relation between a desired level of risk measure accuracy and required sample size under the constraints of computing time and memory. A sensitivity analysis of parameters is further conducted, and our numerical investigations provide practical considerations for insurers in meeting certain regulatory requirements.

Keywords: Affine models; Moment-based method; Risk measures; Correlated risk factors; Insurance with option-embedded features

1. Introduction

Risk measurement is an important component of business insurance and examines the insurer's capability in fulfilling its future obligation once a product is sold. Various kinds of risk measures have emerged in the last few decades, each with certain desirable features; see Balbas, et al. [4], Sereda, et al. [29], and Wirch and Hardy [35], amongst others. Value at risk (VaR), first introduced by Markowitz [23] and Roy [28], stands out amongst many competing risk measures due to its simple implementation in practice. But, its inability to preserve the sub-additivity property is its major drawback. The concept of a coherent risk measure was then proposed by Artzner, et al. [3] to rectify the deficiency of VaR. A representative of coherent risk measures, conditional tail expectation (CTE) has been commonly used in recent years as the alternative to VaR. In Canada's life insurance regulatory framework, the Office of the Superintendent of Financial Institutions Canada requires insurers to use CTE over one year for the supervisory target level (cf. [24]).

Wang, et al. [34] developed a class of risk measures called distortion risk measures which demonstrate the theoretical results of coherent risk measures. These measures are defined by distortion functions such as the

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http://dx.doi.org/10.1016/j.matcom.2016.07.003

proportional hazard (PH) function [32], lookback function (LB) [15] and Wang Transform (WT) function [33]. Wirch and Hardy [35] proved that the distortion risk measures are coherent if and only if the distortion function is concave for positive losses; then, came the introduction of the Beta function as a distortion function. The theory of spectral risk measure (SRM) was developed by Acerbi [1,2] in which this type of risk measure is linked to the user's risk aversion.

Although many risk measures were put forward, there is no consensus which one is the best for risk management. Sortino and Satchell [30] concluded that there is no single risk measure that is universally acceptable because any proposed risk measure would have its own limitations. Rachev, et al. [27] argued that an ideal measure does not exist but it is reasonable to search for risk measures ideal for the specific problem under investigation.

The insurance industry standard keeps evolving in response to new economic conditions and in an effort to set high levels of safety and effectiveness. Given growing uncertainties nowadays, regulatory authorities and entities with oversight functions require higher levels of safety via capital requirements to address companies' insolvency issues. The emergence of contracts like unit-linked life insurance contingencies with guaranteed minimum payoffs entails cautious risk assessment given the interaction of several risk factors.

When option-embedded insurance products began appearing in the market, various papers immediately dealt with its valuation; for example, Ballotta and Haberman [5], Boyle and Hardy [7], Liu, et al. [19], amongst others. The focus of this paper is the estimation of extreme losses that may cause solvency problem for companies. Equitable Life taught us a valuable lesson on the importance of assessing solvency capital adequately. For a long time Equitable Life held small reserves to cover against adverse events due to high interest rates. However, when interest rates fell along with the unanticipated mortality improvement Equitable had to put itself up for sale and close new businesses in 2000. Many insurance companies still use classical methods to evaluate their risks, which cannot offer provisions against capital solvency.

In this paper, we aim to evaluate the capital requirement of GAO through the above-mentioned risk measures and construct a relationship among these risk measures. To attain these objectives, we develop a framework to model the loss (profit is viewed as negative loss) of GAO in which the risk factors are stochastic and correlated with each other. Since it is not easy to identify the underlying distribution of the loss exactly, we adopt the Monte-Carlo simulation to get the approximate empirical distribution and obtain the estimates of the risk measures. However, it is known that the sample variability limits the applications of Monte-Carlo method. That is, different values are obtained under different sample paths. To address the accuracy and credibility of the estimated risk measures, we employ the bootstrap method to estimate the variation. Through regression methodology, we also determine in advance the number of sample replicates needed to achieve the target sampling error and vice versa.

As an alternative approach, we employ the methodology of density approximation to estimate the distribution of the loss random variable. We address the problem of finding the underlying unknown probability distribution function (PDF) and the corresponding cumulative density function (CDF) of the population given samples drawn from a population. Various techniques can be utilised for density approximation; see for instance, Baron and Sheu [6], Devroye and Györfy [10], amongst others. Traditionally, there are two principal approaches for density estimation: parametric, which makes stringent assumptions about the density; and nonparametric, which is essentially a distribution-free approach. Kernel density estimation is one of the most widely-used non-parametric techniques to model densities because it provides a flexible framework to represent multi-modal densities. However, the requirements of high memory and computational complexity limit its applicability in practice. Moment-based density approximation, on the other hand, is an easier way to approximate the density when the moments of a given distribution are available; see Provost [26]. In this work, we utilise the moment-based method to approximate the distribution of the GAO losses given the samples generated from the expression of the loss random variable. We compare this approach with the commonly used non-parametric kernel density estimation, which is executed using the function *density* provided in the software **R** with a Gaussian kernel and the bandwidth is the standard deviation of the smoothing kernel.

This paper is structured as follows. We construct the modelling framework of the correlated risk factors and evaluate the loss of a GAO in Section 2 under the assumption of no charges. Section 3 presents some well-known risk measures with an elaboration of their attractive properties and limitations. In Section 4, we illustrate numerical results covering three related aspects. Firstly, we provide numerical risk measures from empirical CDF and from approximated distributions through moment-based density approximation method. Secondly, the accuracy of the risk measures is examined through the bootstrap method. The byproduct of this process is that we could obtain the number of replicates for a given desired standard error according to a relation derived using a regression method. Finally, a