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Design of unsupervised fractional neural network model optimized with interior point algorithm for solving Bagley–Torvik equation

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Abstract

In this article, an efficient computing technique has been developed for the solution of fractional order systems governed with initial value problems (IVPs) of the Bagley–Torvik equations using fractional neural networks (FNNs) optimized with interior point algorithms (IPAs). The strength of FNNs is exploited to develop an approximate model of the equation in an unsupervised manner. The training of optimal weight of the networks is carried out using IPAs. The designed scheme is evaluated on different IVPs of the equation. Comparative studies for the results of the proposed scheme are made with an available exact solution, Podlubny numerical techniques, an analytical solver based on He's variational iteration method and a reported solution of stochastic solvers based on hybrid approaches, in order to verify the correctness of the design scheme. The results of statistical analysis based on the sufficient large number of independent runs established the consistency of the proposed scheme in terms of accuracy and convergence.

Keywords: Bagley–Torvik equation; Fractional differential equations; Interior point algorithm; Neural networks; Unsupervised modeling

1. Introduction

In the present study, initial value problems of the Bagley–Torvik equations have been taken, to be solved by an efficient soft computing approach based on unsupervised fractional neural networks optimized with interior point methods. The generic form of the Bagley–Torvik equations with initial conditions can be written as

$$AD^2u(x) + BD^{\nu}u(x) + Cu(x) = g(x), \quad 0 < x \leq X, \tag{1}$$
$$D^k u(0) = c_k, \quad k = 0, 1,$$

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where D is the operator that gives the derivative with respect to x, ν is the order of the fractional derivative term in the equation and its value is taken as 1.5 or 1.75, $u(x)$ is the solution of Eq. (1), A, B, and C are given constant coefficients of the equation, and c_k are the constants representing the value of the initial conditions.

The differential equation of fractional order as given in (1) was originally introduced by Torvik and Bagley themselves for their pioneering work on describing the motion of an immersed plate in a Newtonian fluid [6,5]. The detailed descriptions, history, and the importance of Bagley–Torvik equations can be found in [7,8]. The research community has shown great interest in the Bagley–Torvik equations with a 1/2 or 3/2-order fractional derivative term that describes the motion of physical systems, a rigid plate immersed in a Newtonian fluid and a gas in a fluid. These equations also play a fundamental part in modeling of frequency-dependent damping materials [6,5,7,8,27,47].

The numerical and approximate analytical solution for Bagley–Torvik fractional differential equations has been reported by many authors. For example, Podlubny's successive approximation method [27], Haar wavelet operational matrix approach [47], Adomian decomposition method [17,48], collocation–shooting method [3], Taylor collocation method [10], He's variational iteration method [16], differential transform method [4], the matrix approach to discretization of fractional derivatives [28], Adams predictor and corrector approach [11] *etc*. Recently, a number of articles have been published for solving different variants of initial value problems (IVPs) of Bagley–Torvik equations numerically; this is seen in [14,20,21,25,50]. In addition to those, preliminary studies on stochastic numerical solvers based on artificial intelligence or soft computing algorithms using neural networks optimized with genetic algorithms (GAs) and particle swarm optimization (PSO) algorithms [40,42] are also applied to solve the Bagley–Torvik equations, but their lower accuracy and higher number of computations make them more inefficient. However, these studies open a wide alternate research platform to investigating the development of computational intelligence algorithms that overcome such limitations. This also acts as motivation for authors to explore this domain and develop a new soft computing solver to achieve these objectives.

The strength of artificial neural networks (ANNs) is well established for effective and reliable modeling of applied problems arising in diverse fields. For example, controlling a water bath temperature system [26], solving quadratic problems with equality and inequality constraints [54], anti modeling the ward atmosphere in a medical unit [49], an accelerated-time simulation of car traffic on a motorway [1], modeling of thin film flow problems of third grade fluid [35] and an accelerated-time simulation of baggage traffic in an airport terminal [2], *etc*. Strength of approximation theory based on ANNs is used extensively for accurate solutions of initial and boundary value problems (BVPs) of differential equations [45,34,55]. Potential applications of these methodologies are the solving of BVPs of nonlinear functional differential equations of pantograph type [30], Troesch's problems arising in Plasma physics modeling [31,29], fuel ignition models in combustion theory [32,33], and computational fluid mechanics problems based on Jeffery–Hamel flow equations in the presence of a high magnetic field [43,44], *etc*. Use of ANN models for solving linear and nonlinear fractional differential equations has been reported recently in [39,38]. In particular, the modeling of fractional order systems represented by fractional Riccati differential equations and Basset problems is another illustrative example of ANN methodology [37,41,36].

In this paper, an approximate mathematical model for Bagley–Torvik equations is formulated with the help of ANNs in an unsupervised manner. The training of weights for ANNs is made by the interior point algorithm (IPA), used as an efficient local search method for solving constraint optimization problems. A sufficiently large number of simulations of the proposed scheme is carried out in different cases of the Bagley–Torvik equations, validating the effectiveness of the approach. The comparative studies of the results are provided with an available exact solution, numerical method with Podlubny [27,28], and reported results of an approximate analytical solver based on He's variational iteration method (HVIM) [16] and stochastic numerical techniques based on neural networks optimized with hybrid schemes of GA-PS [40] and PSO-PS [42] algorithms.

Organization of the paper is as follows: In Section 2, the basic definitions and important relations used in the rest of the study are introduced. In Section 3, the detail on neural networks mathematical modeling of the Bagley–Torvik equations and formulation of its architecture and fitness function are described. In Section 4, the methodology used for learning of weights of networks with IPA is presented in terms of a step by step procedure and flow diagrams. In Section 5, numerical experimentation of the proposed scheme on two cases of three IVPs of the Bagley–Torvik equations is provided. In Section 6, comparative studies of the proposed scheme are presented based on statistical analysis. In the last section, findings are summarized along with future research directions in this domain.