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Multi-scale modeling of flow resistance in granular porous media

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Abstract

A new methodology for modeling fluid flow through porous beds consisting of spherical or quasi-spherical particles is proposed. The proposed approach integrates the Discrete Element Method (DEM) with Computational Fluid Dynamics (CFD) techniques. A porous bed was first modeled using the DEM to quantify its pore structure, including the number, coordinates and diameters of all spheres in the bed. Then, a numerical algorithm (PathFinder) is developed to use the DEM simulation results to calculate the micro-scale properties of the porous bed, including porosity, tortuosity and other geometrical parameters. Finally, the determined property parameters are used in a CFD model to simulate fluid flow through porous beds. It was shown that the Discrete Element Method was capable of simulating the spatial structure of granular porous media, providing geometrical parameters that are required by commonly used models for predicting flow through porous media.

Keywords: PathFinder code; Porous beds; Spatial structure; Discrete Element Method; Computational Fluid Dynamics

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1. Introduction

Modeling fluid flow in porous media has been an active research field since Darcy proposed his empiric one-dimensional formula in 1856. Two distinguishable approaches are used in modeling flow through porous media. In the macro-scale approach, the porous medium is treated as a homogeneous mass and only the global effect is analyzed, while some fundamental physical characteristics of pore structure are not directly considered. Simple laws and global indicators representing the macroscopic features of porous media describe flow resistance of porous media. The Darcy law [17], Forchheimer law [8], Ergun law [20], and Kozeny–Carman [33,14] law are examples of macro-scale approach, which is relatively simple in terms of calculation, and produces satisfactory results if appropriate parameter values are available. Taking the Forchheimer law as an example, critical parameters are the permeability coefficient and the beta factor, which could be calculated from many formulas available in the literature. However, the differences in results can be drastic when different formula are used [65]. In the macro-scale modeling, two main challenges exist. The first one concerns the scope of application of individual laws, as defined by the Reynolds number. However,

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among researchers there is no agreement on the appropriate values of Reynolds number. Most often it is assumed that the upper limit of Reynolds number for the applicability of Darcy's law is between 1 and 10 [8,15,27,68,69]. Other work indicate the upper limit for Darcy's law as 1 [2], 2 [27], 3 [27] or 5 [27,58]. Some researches suggest that the upper limit goes above 10 [15,27]. The second challenge is obtaining appropriate values of various coefficients, such as permeability coefficient or beta factor mentioned above. As shown in previous studies [65], the best way of obtaining these parameters is to perform a background experiment to measure these coefficients, which may be tedious and time-consuming.

In the micro-scale approach, the porous medium is treated as a system of channels with a particular spatial structure (often simplified), through which the fluid flows. The local effects are analyzed at the level of the individual channels and flow resistance is described by general conservation equations. The micro-scale approach has been used to simulate single and multiphase flows [13,5], non-Newtonian flows [37], non-Darcy flows [6,39,52] reactive transport [29,34], and gas storage [42], as a few examples. The micro-scale modeling involves two main steps. In the first step, the spatial structure of porous medium is determined. This step may be accomplished with the use of simplified geometrical models in which the porous medium is treated as a set of geometrically simple 2D or 3D objects [49,30,40] or with the use of sophisticated geometrical models constructed with imaging techniques (e.g., X-ray) [11,46,71,53]. In the second step, the flow through individual channels within the geometrical models is calculated using CFD techniques. The micro-scale approach is based on the fundamental characteristics of porous media, and thus it can describe physical phenomena occurring in the fluid–solid system better. However, this approach is computationally complex in general and requires special numerical algorithms and high performance computers.

In this article, a novel method of coupling the micro- and macro-approaches in investigations of fluid flows through granular porous media is proposed. The DEM is used to simulate the pore structure of porous media and the PathFinder code is developed to connect the micro-scale to macro-scale [64].

The motivation to develop the new methodology is the observation that many researchers simplify mathematical models due to the difficulty in estimating some key parameters concerning the pore structure of porous media such as the tortuosity. A good example is the work of [62], in which thirteen papers were compared in the context of using the Kozeny–Carman equation. Only in two of them, this equation was described correctly. In other cases, different simplifications were used because the tortuosity, specific surface of the porous body and other pore parameters were not known. With the proposed approach the Kozeny–Carman equation may be used without any simplifications.

2. General form of macro-scale laws

Darcy's law (1856) is widely used to describe flow of fluid through porous media and it has the form of [53,28]

$$-\frac{dp}{dx} = \frac{1}{\kappa}\left(\cdot\mu \cdot \vec{v}_f\right), \tag{1}$$

where: p—pressure [Pa], x—a coordinate along which the pressure drop occurs [m], κ—permeability coefficient [m^2], μ—dynamic viscosity coefficient [kg/(m s)], $\vec{v_f}$—filtration velocity [m/s].

For low velocity flows, Darcy's law adequately describes the flow in porous media [28]. However, as velocity becomes higher, discrepancies between Darcy's law and experimental data appear. Forchheimer [24] attributed this discrepancy to kinetic effects and suggested to add a term $(\rho \cdot \vec{v}_f^{\,2})$ accounting for energy [28,45,21,4] to Darcy's equation:

$$-\frac{dp}{dx} = \frac{1}{\kappa}\left(\cdot\mu \cdot \vec{v}_f\right) + \beta \cdot \left(\rho \cdot \vec{v}_f^{\,2}\right), \tag{2}$$

where β [1/m] is the Forchheimer coefficient (also known as non-Darcy coefficient, or β factor) and ρ is the fluid density [kg/m^3].

Eqs. (1) and (2) may be written together in one general form:

$$-\frac{dp}{dx} = A \cdot \left(\mu \cdot \vec{v}_f\right) + B \cdot \left(\rho \cdot \vec{v}_f^{\,2}\right), \tag{3}$$

where A and B are two generalized parameters which are related to the geometrical structure of the porous medium.