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Global exponential stability of delay difference equations with delayed impulses

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Abstract

This paper investigates the global exponential stability of delay difference equations with delayed impulses. By virtue of Lyapunov functions together with Razumikhin technique, a number of global exponential stability criteria are provided. Both the stability results that impulses act as perturbation and the stability results that impulses act as stabilizer are obtained. Some examples are also presented to illustrate the effectiveness of the obtained results. It should be noted that it is the first time that the Razumikhin type exponential stability results for delay difference equations with delayed impulses are given.

Keywords: Exponential stability; Delay difference equation; Impulse; Lyapunov function; Razumikhin technique

1. Introduction

It is well known that in many evolutionary processes the state may change abruptly at certain moments of time. Consequently, it is natural to assume that these perturbations act instantaneously, that is, in the form of impulses. Many biological phenomena involving threshold, bursting rhythm models in medicine and biology, optimal control models in economics, pharmacokinetics and frequency modulated systems, do exhibit impulse effect. In recent years, the theory of impulsive differential equations has developed rapidly, see [2–4,6,8–10,13,14,16,20,22,25,29,31,32] and the references therein.

The behavior of discrete systems is sometimes extremely different from the behavior of the corresponding continuous systems. Discrete analogs of continuous problems may yield interesting dynamical systems in their own right. Thus many results have been obtained for difference equations/discrete systems, see [1,5,7,11,12,15,17–19,21,23,24,26–28,30,33] and the references therein. However, the theory of impulsive difference equations has developed a little slowly [11,12,17,18,21,23,24,26,28,30,33]. Generally speaking, the stability analysis of impulsive differential equation is not applicable to the impulsive difference equation. Therefore, the detailed analysis for impulsive difference equations is necessary and important.

On the other hand, time delays occur frequently in many evolution processes [5,8,10–15,17,19,21,23–27,29,30,32,33]. Time delays are the inherent features of many physical processes, and they are the big sources of instability

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and poor performances. Thus it is necessary to study the impulsive delay difference equations. However, the theory of impulsive delay difference equations/impulsive delay discrete systems has developed rather slowly due to the difficulty of dealing with the delays and the impulses [11,12,17,21,23,24,26,30,33].

We can easily see that in the previous works about impulsive differential/difference equations, the authors always suppose that the state variables on the impulses are only related to the nearest state variables. But in most cases, it is more applicable that the state variables on the impulses are also related to the time delays, see [8,29]. But there are rare results about this kind of impulsive delay difference equations.

Motivated by the above discussions, in the present paper, we will consider the global exponential stability of delay difference equations with delayed impulses. When studying delay difference equations with impulses, the hardest thing is to how to deal with the delays and the state variables on the impulses. The Razumikhin technique is one of the useful methods to investigate the stability of systems with time delays. The Razumikhin technique has advantage that, when dealing with time delays, the Lyapunov function is not required to be decrescent on the whole state space [19]. The Razumikhin approach has a lower complexity when the size of the delay increases and it applies directly to the nonaugmented system [5], in general, such approach is easier to apply [27]. In [33], the authors studied the global exponential stability of a class of impulsive delay difference equations, by using the properties of "ρ-cone" and eigenspace of the spectral radius of nonnegative matrices, some results have been given. While in the present paper, we will consider the global exponential stability of impulsive delay difference equations in general form. We will use Lyapunov functions together with Razumikhin technique to obtain some new results. It should be noted that even time delays do not exist in the state variables of the impulses, some of the results we obtained can still be used to study the stability of some impulsive delay difference equations that cannot be studied by [11,12,23,33]. In addition, one of the examples shows that, though some delay difference equations without impulses are unstable, they may become stable if appropriate impulses are added to them. That is, in some cases, impulses play a dominating part in causing the stability of the delay difference equations.

This paper is organized as follows. In Section 2, some basic definitions and notations are introduced. In Section 3, some global exponential stability criteria for delay difference equations with delayed impulses are provided. In Section 4, some examples are presented to illustrate the effectiveness of the proposed results. Finally, concluding remarks are given in Section 5.

2. Preliminaries

Consider the following delay difference equations with delayed impulses.

$$\begin{cases} x(n+1) = f(n, x_n), & n \geq 0, n \neq n_k - 1; \\ x(n_k) = I_k(x(n_k - \theta(k))), & k = 1, 2, \ldots; \\ x(s) = \varphi(s), & s \in J \end{cases} \tag{1}$$

where $x \in R^m$, $x_n(s) = x(n+s)$ for $s \in J = \{-\tau, -\tau+1, \ldots, -1, 0\}$, $f \in C(N \times C(J, R^m), R^m)$, N denotes the set of nonnegative integers, $I_k \in C(R^m, R^m)$, $I_k(0) \equiv 0$, $\varphi \in C(J, R^m)$, $\tau \in Z^+$, Z^+ denotes the set of positive integers, $\theta(k) \in Z^+$ and $\sup_{k\in Z^+}\{\theta(k)\} = \bar{\theta} \leq \tau$, $f(n, 0) \equiv 0$, $n_k \in Z^+$ for $k \in Z^+$, $0 = n_0 < n_1 < n_2 < \cdots < n_k < \cdots$, $n_k \to \infty$ for $k \to \infty$, $\tau > 0$. Let $\|\cdot\|$ denote Euclidean norm for vectors, $|\phi| = \max_{s\in J}\{\|\phi(s)\|\}$.

Obviously, $x(n) = 0$ is a solution of (1) which we call the zero solution. We assume that Eq. (1) has a unique solution, denoted by $x(n) = x(n; 0, \varphi)$, for any given $\varphi \in C(J, R^m)$.

We have the following definition.

Definition. The zero solution of (1) is said to be globally exponentially stable, if there exist scalars $\gamma > 0, \sigma \geq 1$ such that

$$\|x(n; 0, \varphi)\| < \sigma|\varphi|e^{-\gamma n}, \quad n \geq 0.$$

3. Main results

Now we consider the impulsive delay difference equations (1), we have the following results.