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Inference and prediction for modified Weibull distribution based on doubly censored samples

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Abstract

In this article, inference for the modified Weibull (MW) distribution under type-II doubly censored sample is discussed. Maximum likelihood estimator (MLE) and Bayes estimators (BEs) based on conjugate and discrete priors are derived for three unknown parameters. The BEs are studied under squared error loss and LINEX error loss functions. The Bayesian prediction (BP) of the ℓ-th ordered observation x_ℓ in a sample of size n from MW distribution is obtained. A real life data set and simulation data are used to illustrate the results derived.

Keywords: Bayesian estimation; Bayesian prediction; Type-II double censoring; Three-parameter modified Weibull distribution; Order statistics

1. Introduction

The modified Weibull is an important distribution in modeling lifetime data because of its flexibility to fit different types of data. Also, it contains several important distributions such as Weibull, linear exponential, and Rayleigh distributions.

We assume that the lifetime X of a product follows the MW distribution which is considered by Sarhan and Zaindin [32] with probability density function (pdf)

$$f(x;\alpha,\theta,\beta)=\left(\alpha+\theta\beta x^{\beta-1}\right)\exp\left(-\alpha x-\theta x^{\beta}\right),\quad x>0,\ \alpha,\ \theta\geq 0,\ \beta>0, \tag{1}$$

where α is a scale parameter and θ and β are shape parameters such that $\alpha+\theta>0$. The cumulative distribution function (cdf) is given by

$$F(x;\alpha,\theta,\beta)=1-\exp\left(-\alpha x-\theta x^{\beta}\right),\quad x>0. \tag{2}$$

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be order statistics from a sample of size n with pdf and cdf defined in (1) and (2), respectively. Suppose that $\mathbf{X}=(X_{(r)}, X_{(r+1)}, \ldots, X_{(s)})$, $1\leq r\leq s<n$ be type-II doubly censored sample. When

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$r = 1$, the type-II doubly censored sample becomes the well-known type-II right censored sample. The likelihood function (LF) of $\vartheta = (\alpha, \theta, \beta)$ for the above censored sample is given by

$$L(\vartheta|\mathbf{X}) = \frac{n!}{(r-1)!(n-s)!}[F(x_r)]^{r-1}[1-F(x_s)]^{n-s}\prod_{j=r}^{s} f(x_j). \tag{3}$$

The predictive likelihood function of ϑ and x_ℓ for type-II doubly censored sample is accordingly given by

$$L(x_\ell, \vartheta|\mathbf{X}) = \begin{cases} \dfrac{n!}{(\ell-1)!(r-\ell-1)!(n-s)!}[F(x_\ell)]^{\ell-1}[1-F(x_s)]^{n-s}\prod_{j=r}^{s} f(x_j) \\ \quad \times [F(x_r)-F(x_\ell)]^{r-\ell-1} f(x_\ell), \quad \ell = 1,\ldots,r-1, \\ \dfrac{n!}{(r-1)!(n-\ell)!(\ell-s-1)!}[F(x_r)]^{r-1}[1-F(x_\ell)]^{n-\ell}\prod_{j=r}^{s} f(x_j) \\ \quad \times [F(x_\ell)-F(x_s)]^{\ell-s-1} f(x_\ell), \quad \ell = s+1,\ldots,n. \end{cases} \tag{4}$$

Some articles on type-II doubly censored sample problems are found in Fernández [13,12], Kambo [20], Kim and Song [22], Raqab [27], Raqab and Madi [29], Sarhan [31] and among others. Bayesian prediction bounds have been discussed extensively by many authors, see for example Abd El-Aty et al. [1], Al-Hussaini and Ahmad [5], Ali Mousa [6], Geisser [15,16], Jaheen [18], Kundu and Raqab [24], Mohie El-Din et al. [26] and Raqab [28]. Maximum likelihood and Bayes estimators have also been discussed by a number of authors, including Ahmad et al. [3], Balakrishnan and Chan [8], Balakrishnan and Cohen [9], Basak et al. [10], Calabria and Pulcini [11], Raqab and Madi [30] and Singh et al. [34]. Kotb [23] dealt with Bayesian inference and prediction for MW distribution based on generalized order statistics. There is some research regarding Bayesian and non-Bayesian estimations of the MW distribution, see for example, Al-Omari and Al-Hadhrami [7], Gasmi and Berzig [14] and Zaindin and Sarhan [37].

The rest of the paper is organized as follows: In Section 2, we present Bayes and maximum likelihood estimations to estimate the model parameters. In Section 3, we use the BP (point and interval) and alternative BP (point and interval) in the case of one-sample scheme of the MW distribution based on type-II doubly censored sample. A real life data set and simulation data are used to illustrate the theoretical results in Section 4. Finally, we conclude with some remarks and a brief summary of the results in Section 5.

2. Estimation of model parameters

In this section, we describe BEs and MLE of α, θ and β. Using (1)–(3), the likelihood function (LF) can be written as

$$L(\vartheta|\mathbf{X}) = \frac{n!}{(r-1)!(n-s)!}\left(\prod_{j=r}^{s}\left(\alpha+\theta Z_j(\beta)\right)\right)\sum_{i=0}^{r-1} c_{i,r}\exp\left[-\left(\alpha\psi_i(1)+\theta\psi_i(\beta)\right)\right],$$

where $Z_j(\beta) = \beta x_j^{\beta-1}$, $\psi_i(\beta) = i x_r^\beta + (n-s)x_s^\beta + \sum_{j=r}^{s} x_j^\beta$ and $c_{i,r} = (-1)^i\binom{r-1}{i}$.

Making use of the identity (Hardy et al. [17])

$$\prod_{j=r}^{s}\left(\alpha+\theta Z_j(\beta)\right) = \sum_{j=0}^{s-r+1}\alpha^{s-r-j+1}\theta^j\xi_j(\beta), \tag{5}$$

where for $j = 0$, $\xi_0(\beta) = 1$ and for $r \le j \le s$,

$$\xi_j(\beta) = \sum_{b_1=r}^{s-j+1} Z_{b_1}(\beta)\sum_{b_2=b_1+1}^{s-j+2} Z_{b_2}(\beta)\times\cdots\times\sum_{b_j=b_{j-1}+1}^{s} Z_{b_j}(\beta),$$

we can rewrite the LF in the following form

$$L(\vartheta|\mathbf{X}) = \frac{n!}{(r-1)!(n-s)!}\sum_{i=0}^{r-1} c_{i,r}\sum_{j=0}^{s-r+1}\xi_j(\beta)\alpha^{s-r-j+1}\theta^j\exp\left[-\left(\alpha\psi_i(1)+\theta\psi_i(\beta)\right)\right].$$