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Chaos control for a unified chaotic system using output feedback controllers

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Abstract

This paper presents a new method of controlling a unified chaotic system by using output feedback control strategy. In particular, for an arbitrarily given equilibrium point of a unified chaotic system, we design explicit and simple output feedback control laws by which the equilibrium point is globally and exponentially stabilized. Computer simulations are employed to illustrate the theoretical results.

Keywords: A unified chaotic system; Output feedback control; Equilibrium point; Globally asymptotical stability

1. Introduction

Chaotic system is a complex dynamical nonlinear system and its response exhibits some specific characteristics such as excessive sensitivity to initial conditions, broad Fourier transform spectrums, and irregular identities of the motion in phase space [4,10,17,18]. Also, it has been found to be useful in analyzing many problems, such as information processing, power systems collapse prevention, high-performance circuits and devices, etc.

In 1963, Lorenz presented the first classical chaotic system [10] in a third-order autonomous system with only two multiplication-type quadratic terms but the system displays very complex dynamical behaviors. Chen and Ueta found another chaotic system in 1999, the Chen system, which is similar, but topologically non-equivalent to the Lorenz system [4]. By the definition of Vanecek and Celikovsky [17], the two systems belong to two different types: The Lorenz system satisfies the condition $a_{12}a_{21} > 0$ and the Chen system satisfies $a_{12}a_{21} < 0$, where a_{12} and a_{21} are the corresponding elements in the linear part matrix $A = (a_{ij})_{3\times 3}$ of the system. In 2002, Lü and Chen found another critical system between the Lorenz and Chen systems, bearing the name of the Lü system [11], which satisfies the condition of $a_{12}a_{21} = 0$. To bridge the gap between the Lorenz and Chen systems, Lü et al. introduced a unified chaotic system [12] in the same year, which contains the Lorenz and Chen systems as two extremes and the Lü system as a transition system between the Lorenz and Chen systems [3,12].

For quite a long period, people thought that chaos was neither predictable nor controllable. However, the OGY method [20] developed in 1990s of the last century had completely changed the situation, and then the study of chaos

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control began. The main goal of chaos control was to eliminate chaotic motion and to stabilize one of the system's equilibrium points. Until now, many efficient approaches had been proposed for controlling chaos, such as state feedback control [3], variable structure control [19], generalized OGY control [20], inverse optimal control [14,15], parameter identification control [3], digital control [3], fuzzy control [1,2], adaptive control [3,5], and data sampling control [13], etc. Most of these methodologies need state vectors, while the state vectors are not all measurable in practical application, and thus they are not very useful in practice.

In this paper, based on appropriate Lyapunov functions [6–9], we provide a new method to design explicit and simple output feedback controllers. For an arbitrarily given equilibrium point of a unified chaotic system, we design output feedback controllers to stabilize the equilibrium point globally, with exponential convergence, and provide the Lyapunov negative exponent estimation. This is particularly useful in practical designs. Computer simulations are presented to illustrate the theoretical results.

2. Main results

Consider the unified chaotic system described by [16]

$$\begin{aligned} \dot{x}_1 &= (25\alpha+10)(x_2-x_1) \\ \dot{x}_2 &= (28-35\alpha)x_1+(29\alpha-1)x_2-x_1x_3 \\ \dot{x}_3 &= x_1x_2-\frac{8+\alpha}{3}x_3 \\ y &= x_2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are state variables, y is the output variable, and $\alpha \in [0, 1]$. Obviously, system (1) becomes the original Lorenz system for $\alpha = 0$; while system (1) becomes the original Chen system for $\alpha = 1$. When $\alpha = 0.8$, system (1) becomes the critical system. In particular, system (1) bridges the gap between Lorenz system and Chen system. Moreover, system (1) is always chaotic in the whole interval $\alpha \in [0, 1]$. It is easy to find the three equilibrium points of the system:

$$E_1 = (0, 0, 0), \qquad E_2 = (\beta, \beta, \gamma), \qquad E_3 = (-\beta, -\beta, \gamma)$$

where $\beta = \sqrt{(8+\alpha)(9-2\alpha)}$ and $\gamma = 27 - 6\alpha$.

Denoting the above equilibrium points as (x_1^0, x_2^0, x_3^0), the following transformation is introduced:

$$\begin{aligned} e_i &= x_i - x_i^0, \quad (i = 1, 2, 3), \\ e_y &= y - x_2^0 \end{aligned} \tag{2}$$

and then by adding output feedback controls to system (1), the following control system can finally get yielded:

$$\begin{aligned} \dot{e}_1 &= (25\alpha+10)(e_2-e_1)+u_1(e_y) \\ \dot{e}_2 &= (28-35\alpha-x_3^0)e_1+(29\alpha-1)e_2-e_1e_3-x_1^0e_3+u_2(e_y) \\ \dot{e}_3 &= -\frac{8+\alpha}{3}e_3+e_1e_2+x_2^0e_1+x_1^0e_2 \\ e_y &= e_2 \end{aligned} \tag{3}$$

where $u_i(e_y)$ are linear control functions satisfying $u_i(0) = 0$, $(i = 1, 2)$.

Before proceeding further, we will state a well-known lemma as follows.

Lemma 1. *For any real scalar $\epsilon > 0$ and any vectors with appropriate dimensions a and b, the following inequality holds:*

$$2|ab| \le \epsilon a^2 + \epsilon^{-1} b^2.$$

Theorem 1. *The following control:*

$$\begin{aligned} &u_1 = k_1 e_y, k_1 = -(10+25\alpha), \\ &u_2 = k_2 e_y, k_2 = 1-29\alpha-\mu, \quad (\mu > 0) \end{aligned} \tag{4}$$

can be used to stabilize the equilibrium point (x_1^0, x_2^0, x_3^0) globally, where μ is a given parameter satisfying $\mu > 0$.