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Approximation of the frequency response of power systems based on scale invariance

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Abstract

Power networks are complex systems composed of many heterogeneous and interacting components. Smart grids are even more complex systems due to the convergence of electrical and communication networks. In order to deal with this complexity, a mathematical model that is reduced-size, accurate, wide-band and knowledge based is required for dynamic studies. This paper introduces a novel modeling approach based on scale invariance to build an approximation of the frequency response of power systems. This approach combines an asymptotic and a resonant model. Both use the spectral dimension of the network which is a key parameter to describe its scale invariance. The resonant model is identified by using an improved vector fitting method. The improvement consists in a guess of the initial poles used for the identification which is deduced from the scale invariant distribution of the dynamic modes of the network. An application to an IEEE test transmission system is finally shown.

Keywords: Scale invariance; Power law; Frequency response; Power systems; Vector fitting

1. Introduction

Smart grids interconnect power systems and ICT (information and communication) networks for a better integration of renewable energy sources and fulfilling customers' needs of a safe, reliable, competitive and efficient electrical energy. It consists in the implementation of new functions of analysis, control, and monitoring [2]. Smart grids are complex systems because of their hierarchical and multilayered structure, the numerous dynamic interactions between the multiple layers and the highly distributed components, their possible chaotic behavior such as blackout [15].

Co-simulating smart grids has become a mandatory step toward the resolution of problems raised by the optimal design and implementation of new functions of management of such complex systems [17,19]. However, while power systems are modeled by continuous equations, a communication system has discrete event dynamics. Generally, co-simulation platforms combine several simulators into a federation. Each simulator is dedicated to a part of the smart grid. Agent-based frameworks can be used for combining the simulators or for simulating specific entities of the smart

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grids [10,22]. Of course, co-simulation platforms cope with computational constraints and reduced-size models are preferred to decrease the associated costs.

The complexity makes difficult the search of parsimony, especially in the dynamic domain. The power engineers and researchers cope with a double challenge. The size of the models of the smart grids shall be reduced while a maximum of details shall be kept about the dynamic interactions. Indeed, smart grids are highly distributed systems and their internal dynamic couplings cannot be ignored if the purpose is to increase their stability and controllability [5].

In order to deal with the increasing size, a *classical approach* is to split the system into two zones: *the study zone* and *the external zone*. The study zone is described in details by dynamic equations, while the model of the external zone is a dynamic equivalent [12,31]. Dynamic equivalent means an approximated representation of the input impedance of the external network "seen" by the study zone. Dynamic equivalents can be classified into three categories depending on the type of dynamic study which is carried on. One distinguishes high-frequency equivalents (HFE), low frequency equivalents (LFE) and wideband equivalents. The first ones are used to study electromagnetic phenomena by representing the frequency-dependent variations of the external zone response. LFE are applied to simulate electromechanical phenomena in the range of 0–2 Hz, such as transient rotor-angle stability of synchronous machines. The wide-band equivalents shall enable the simulation of either low dynamic responses for control purposes or high dynamic transient operations for protection design. Of course, dynamic equivalents are expected to be as accurate as possible, reduced-size, and knowledge-based. The reduced size shall decrease the computation burden and the knowledge-based characteristic shall facilitate the modeling and analysis work by engineers. Dynamic equivalents can be adopted for off-line or real time simulation [12].

The reduction techniques used to obtain classical dynamic equivalents lead to a loss of physical meanings about the dynamics interactions within the power systems. Indeed, the size of the model is decreased by canceling some dynamic modes. To proceed to this cancellation, aggregation or identification techniques are used. Aggregation means that all the dynamics of the external part are aggregated into a more simple system. For instance, for low frequency equivalents, the generators connected to the external grid can be aggregated into a single aggregated generator whose dynamic characteristics represents the average dynamic of the initial generators. The identification method consists of identifying the dynamic response of the external part by an approximated model whose mathematical structure is given a priori. Of course, if this approach can give very accurate results, it leads to a black-box model, meaning that the internal structure of the model does not have any physical sense.

The main objective of this paper is to reintroduce some physical knowledge into the mathematical representation of the external part while keeping its size controlled. Hence, we propose to develop a dynamic equivalent based on the concept of *scale-invariance* with the aim of coping with the complexity of smart grids. As many complex systems, smart grids are expected to be scale invariant because of the self-similarities of their hierarchic structure [27,28]. Scale invariance means that some patterns are repeated in the hierarchical arrangement of the different levels of the smart grid. Scale invariance shows itself by power-law relations which appear between different levels of observation of the system. A crucial parameter to characterize the scale invariance is the power-law exponent [3]. Power-law functions have already been used for power systems in [4] and for ICT infrastructure in [7]. Usually, power-law functions are used for characterization purposes and the value of their exponent helps to sort out the topological properties of networks. In the present paper, *scale invariance will be introduced into the dynamic equivalent to control its size while representing the dynamic couplings inside the smart grid*.

This paper presents an unconventional and innovative approach to obtain equivalent dynamic model of smart grids. Of course, the authors do not pretend to be able today to solve the complete problem. So, the aim of this paper is first to give the basis of our approach and shows its relevancy. For that reason, the presented works will focus on *high frequency equivalents of power systems* only.

These equivalents are generally obtained by the identification of an approximation of the frequency response of the Norton admittance of the external network seen from its point of connection with the study zone [18]. The paper will aim at introducing scale-invariant properties into this approximation.

As previously mentioned, this paper will focus on the power systems only. Even if our work is expected to be extended in the future to ICT networks, the preliminary developments have been based, for simplicity purpose, on the continuous equations describing the electrical grids.

Then, the first section of the paper refers to the characterization of the scale invariance of power systems. The second part details the scale invariance properties of fractal synthetic grids and realistic power systems. The third and