Available online at www.sciencedirect.com
ScienceDirect
Mathematics and Computers in Simulation 131 (2017) 295–315
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original articles

Modelling of a beam excited by piezoelectric actuators in view of tactile applications

Clément Nadal*, Christophe Giraud-Audine, Frédéric Giraud, Michel Amberg, Betty Lemaire-Semail

L2EP–IRCICA, Université de Lille 1, Parc Scientifique de la Haute Borne, 50 avenue Halley–BP 70478, 59658 Villeneuve d'Ascq, France

Received 16 October 2014; received in revised form 24 September 2015; accepted 30 October 2015
Available online 22 November 2015

Abstract

This paper deals with a one-dimensional semi-analytical modelling of the vibratory behaviour of a rectangular beam excited by several piezoelectric ceramics glued on its lower face. The establishment of the equations of motion is secured by the application of Hamilton's principle. From this approach, it results an accurate knowledge on the mode shape in function of the geometry, the structure, and the positions of the piezoelectric actuators. This approach deployed on a simple case falls within a general process of tactile surfaces optimization. This article shows that there is a trade-off between the optimization of the homogeneity of the tactile stimulation and the electromechanical coupling factor.

Keywords: Semi-analytical modelling; Piezoelectricity; Tactile applications; Hamilton's principle

1. Introduction

During the last decade, a noticeable interest has been shown in the sense of touch in the human–computer interaction, owing to its importance in the perception of our world by the manipulation and identification of the objects surrounding us. In order to enhance the interaction between the user and the communicating device, it is considered to provide a rendering of tactile sensations for the former matching the actions he performs. In this sense, the field of tactile feedback has been emphasized by the emergence of several technologies. One of them uses lateral forces which create the illusion of fine textures and surface features such as the skin or the textile [12]. Many haptic surfaces have been realized to take advantage of this illusion. Watanabe and Fukui [11] developed the first ultrasonic vibrating plate, with a resonant frequency of 75.6 kHz and a 2 µm amplitude, able to control the roughness surface displayed to a bare finger. The main idea was to use the reduction of the effective friction force thanks to an air film between the vibrating plate and user's fingertip, the so-called *squeeze film effect*. Biet et al. [2] used a structure composed of an array of piezoelectric actuators bonded on the lower face of a metallic plate to generate out-of-plane vibrations of a 1 µm

* Corresponding author. Tel.: +33 0 3 62 53 16 32; fax: +33 0 3 20 43 69 67.
E-mail addresses: clement.nadal@ircica.univ-lille1.fr (C. Nadal), christophe.giraud-audine@ensam.eu (C. Giraud-Audine), frederic.giraud@polytech-lille.fr (F. Giraud), michel.amberg@univ-lille1.fr (M. Amberg), betty.semail@polytech-lille.fr (B. Lemaire-Semail).

http://dx.doi.org/10.1016/j.matcom.2015.10.011

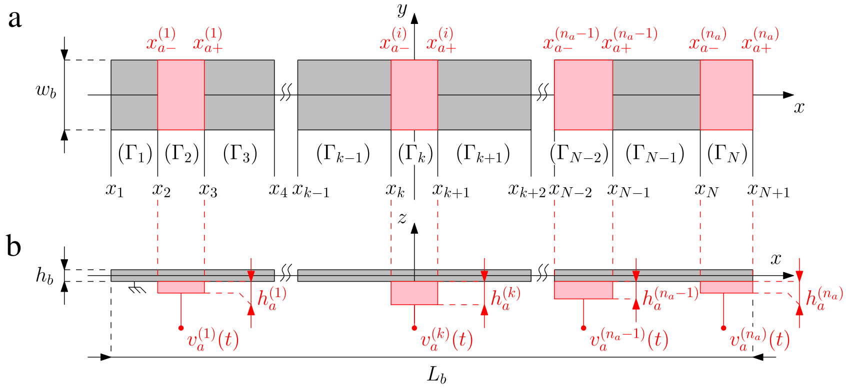

Fig. 1. Bottom view (a) and side view (b) of a beam excited by n_a piezoelectric actuators.

amplitude at a 30.5 kHz resonant frequency. A circular variant of such a device was developed by Winfield et al. [13] who mounted a glass disk on a circular piezoelectric actuator. In a near future, these technologies will be destined to equip general public systems with tactile screen (smart phones, tablet computers, laptops, remote controls, etc.). In fact, their implementation in a limited environment gives rise to optimization questions in terms of, especially, energy consumption and mass of used piezoelectric ceramics. A first attempt has been carried out by Sergeant et al. [10] who determined the optimal dimensions of a resonator and the ceramics composing the device in order to obtain a maximal deflection with a minimal supply voltage.

In this paper, a different problem is addressed. Indeed, it is shown [2] that a homogeneous deformation is required in order to get a uniform tactile stimulus. Thus, the goal is to obtain such a condition with a minimal amount of ceramics and enough amplitude. Moreover, the location of the piezoelectric ceramic actuators is also optimized to select exact mode shapes to realize a satisfactory stimulation. Briefly put, deformation should be as homogeneous as possible in order to obtain a uniform tactile stimulus all over the plate surface while having the largest electromechanical conversion factor.

To achieve this, a semi-analytical model is developed in this paper. Its main aim is to study the influence of the ceramics on the resonant frequencies (the squeeze film effect being effective for a frequency greater than 25 kHz [2]) and mode shapes, especially in terms of promotion and uniformity. Based on some well-known assumptions used for the study of laminated structures, such a modelling is commonly carried out for active vibration control. For instance, Ducarne et al. [3] optimize the placement and dimensions of shunted piezoelectric ceramics dedicated to vibration reduction. Numerical methods have been also validated for the understanding of the dynamic of these active structures (see e.g. [7]). In this paper, a generalized model of a free–free beam excited by n_a piezoelectric actuators bonded on its lower faces, defining a non-symmetric geometry, is detailed. In fact, in this article, the geometry and different assumptions enabling its formulation will be firstly specified. Then, the equations of motion will be derived from the application of Hamilton's principle. Finally, example structures consisted of two ceramics will be simulated and studied in terms of homogeneity and promotion of mechanical mode shapes.

2. Semi-analytical modelling

2.1. Geometry

The semi-analytical modelling developed below relies on a geometry of a rectangular cross-section beam composed of N parts in the longitudinal direction and excited on its back, as illustrated by Fig. 1, by n_a piezoelectric ceramics behaving like actuators. The beam is supposed to be a parallelepiped of dimensions $L_b \times w_b \times h_b$. The ith actuator is a parallelepiped of the same width than the beam $w_a^{(i)} = w_b$, a length $L_a^{(i)}$ and a thickness $h_a^{(i)}$. These ceramics are polarized along the z-axis and are supplied from their lower and upper faces in order to preferentially use a transverse piezoelectric coupling (variation of the sample length along the x-axis perpendicular to ceramic's polarization direction). Thereafter, the Cartesian coordinate system is chosen and its origin is placed in the centre of the beam. Moreover, the ith ceramic actuator are located by the set of abscissae $(x_{a-}^{(i)}, x_{a+}^{(i)})$.