Engineering Analysis with Boundary Elements 35 (2011) 1123–1135

Contents lists available at ScienceDirect

Engineering Analysis with Boundary Elements

journal homepage: www.elsevier.com/locate/enganabound

A three-dimensional adaptive analysis using the meshfree node-based smoothed point interpolation method (NS-PIM)

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ARTICLE INFO

Article history:
Received 20 March 2009
Accepted 9 May 2011
Available online 7 June 2011

Keywords:
Numerical methods
Meshfree methods
Error indicator
Adaptive analysis
Point interpolation
Tetrahedral mesh generation

ABSTRACT

In this paper, a three-dimensional (3-D) adaptive analysis procedure is proposed using the meshfree node-based smoothed point interpolation method (NS-PIM). Previous study has shown that the NS-PIM works well with the simplest four-node tetrahedral mesh, which is easy to be implemented for complicated geometry. In contrast to the displacement-based FEM providing lower bound solutions, the NS-PIM possesses the attractive property of providing upper bound solutions in strain energy norm. In the present adaptive procedure, a novel error indicator is devised for NS-PIM settings, which evaluates the maximum difference of strain energy values among four nodes in each of the tetrahedral cells. A simple *h*-type local refinement scheme is adopted and coupled with 3-D mesh automatic generator based on Delaunay technology. Numerical results indicate that the adaptive refinement procedure can effectively capture the stress concentration and solution singularities, and carry out local refinement automatically. The present adaptive procedure achieves much higher convergence in strain energy solution compared to the uniform refinement, and obtains upper bound solution in strain energy efficiently for force driven problems.

1. Introduction

It is well known that adaptive analysis is important in the computational mechanics and engineering structural designs. The ultimate objective of adaptive analysis is to achieve desired high accuracy with minimum computational cost. Adaptive mesh refinement techniques along with proper error analysis have been well studied in the traditional FEM [1–4], but much less well-developed for meshfree method [5,6].

The meshfree node-based point interpolation method (NS-PIM or LC-PIM) [7] has been recently developed using the generalized smoothed Galerkin (GS- Galerkin) weak form [8] and PIM shape functions with a set of small number of nodes located in a local support domain. The PIM (polynomial PIM or radial PIM) shape functions possess Delta function property, which allows straightforward imposition of point essential boundary conditions. The use of node-based smoothing domains [7,9–11] provides the so-called "soft" effect to the discrete model and gives a number of good features. Compared with the linear FEM, NS-PIM can obtain better accuracy and higher convergence rate, especially for stress results. Recently, Liu and Zhang [12] made a thorough theoretical study on the NS-PIM and obtained some important conclusions including the upper bound property. NS-PIM is easy to be implemented and suitable for adaptive analysis. An adaptive analysis using the NS-PIM has been conducted to certify solutions with exact bounds of strain energy for 2-D problems [13].

A reliable and efficient error indicator and the associated refinement strategy are crucial issues for the adaptive procedure, especially for 3-D problems. In general, two distinct types of procedures are currently available for deriving error indicators: the recovery based error indicator and the residual based error indicator. The recovery based error indicator was first introduced by Zienkiewicz and Zhu [1] in 1987, by constructing locally an improved solution from the approximation. The recovery processes play a critical role in the computation of this error indicator and there are many papers published on this recovery methodology [2,14,15]. Residual based error indicators make use of the residual of the numerical approximation, either explicitly or implicitly, which offers a very effective alternative [16–20].

Refinement schemes can also be classified into three categories: *h*-refinement, *p*-refinement and *r*-refinement. The *h*-refinement scheme changes the size of element in a localized fashion based on

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doi:10.1016/j.enganabound.2010.05.019

the error indicator. The *p*-refinement scheme is to increase the order of the polynomial, and the *r*-refinement keeps the total number of nodes unchanged but to adjust their position to obtain an optimal approximation [21,22]. Because the NS-PIM works well with lower order interpolation of displacement field, this work chooses to use the *h*-refinement in our adaptive analysis.

In the framework of element-free Galerkin (EFG) method, Chung and Belytschko [23] proposed local and global error indicator for adaptive analysis. Furthermore, Lee and Zhou [24,25] have done intensive works for refinement scheme in EFG. Duarte and Oden [26] derived an error indicator that involves the computation of the interior residuals and the residuals for Neumann boundary conditions for hp-cloud method. Belytschko et al. [27] have used the adaptive analysis based on the residuals for the reproducing kernel particle method (RKPM). Gan et al. [28] presented the adaptive procedure of RKPM for 3D contact problems with elastic–plastic dynamic large deformation. Liu and Tu [29] have introduced an adaptive procedure for meshfree methods using an error indicator of energy error computed via different order of sampling in Gauss integration based on background cells. Rabczuk and Belytschko [30] proposed adaptive analysis for structured meshfree particle methods in 2-D and 3-D problems. Park and Kwon [31] used a least square mesh free method with Voronoi cells to refine interesting regions. Angulo et al. [32] implemented adaptive produce of meshfree finite point method for solving boundary value problems.

In this paper, we propose an efficient error indicator and the associated refinement scheme within the framework of the NS-PIM for adaptive analysis of three-dimensional problems. The new error indicator is defined based on maximum differences of strain energy between the four of each tetrahedral cells connected to the node. A simple *h*-type refinement scheme is then implemented with an effective strategy for adding in nodes into the regions identified by the error indicator. An automatic three-dimensional mesh generator based on Delaunay technology is next coded to generate high quality tetrahedral meshes for each step in the adaptive process. Adaptive analysis is finally performed for a number of 3-D problems, including ones with stress concentration and singularities. The results demonstrate that the present adaptive procedure performed very well for the NS-PIM to obtain solutions of desired accuracy and with upper bounds to the exact solution.

The paper is organized as follows. In Section 2, a brief description and the basic equations of NS-PIM are given. Section 3 describes the effective adaptive procedure, including the definition of error indicator, the calculation of local critical value, the strategy of refinement and the three-dimensional mesh automatic generation. In Section 4, analyses of some 3-D numerical problems are carried out to assess the capabilities of the proposed adaptive procedure. Conclusions are stated in Section 5.

2. Briefing on the NS-PIM

2.1. Basic equations

Consider a solid mechanics problem in three-dimensional domain Ω bounded by $\Gamma(\Gamma=\Gamma_t+\Gamma_u)$. The standard strong form governing equations can be expressed by the following equations [33]:

Equilibrium equation

$$\mathbf{L}^T\boldsymbol{\sigma}+\mathbf{b}=0 \quad \text{in } \Omega, \tag{1}$$

where $\mathbf{L}$ is a differential operator in the following form,

$$\mathbf{L}^T=\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial y} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial z} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}, \tag{2}$$

$\boldsymbol{\sigma}^T=\{\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{xy} \quad \sigma_{yz} \quad \sigma_{zx}\}$ is the vector containing stress component, $\mathbf{u}^T=\{u \quad v \quad w\}$ is the displacement vector, and $\mathbf{b}^T=\{b_x \quad b_y \quad b_z\}$ is the external body force vector.

Essential boundary conditions:

$$\mathbf{u}=\mathbf{u}_p \text{ on } \Gamma_u, \tag{3}$$

where $\mathbf{u}_p$ is the prescribed displacement on the essential boundaries.

Natural boundary conditions:

$$\boldsymbol{\sigma}\cdot\mathbf{n}=\mathbf{t}_p \text{ on } \Gamma_t, \tag{4}$$

where $\mathbf{t}_p$ is the prescribed traction on the natural boundaries, and $\mathbf{n}$ is the vector of unit outward normal on Γ_t.

2.2. Construction of PIM shape functions

PIM shape functions are constructed using a set of small number of nodes located in a local support domain. There are two types of PIM shape functions, which have been developed with different basis functions, i.e. polynomial basis functions [7,9,12,34] and radial basis functions [11,35–38]. In this work we use both the simplest linear polynomial basis functions and effective radial basis functions to construct PIM shape functions.

For the polynomial PIM, the formulations start with the following assumption:

$$u(\mathbf{x})=\sum_{i=1}^{n}P_i(\mathbf{x})a_i=\mathbf{P}^T(\mathbf{x})\mathbf{a}, \tag{5}$$

where $u(\mathbf{x})$ is a field variable function defined in the Cartesian coordinate space $\mathbf{x}^T=\{x \quad y \quad z\}$, $P_i(\mathbf{x})$ is the basis function of monomials, which is usually built utilizing Pascal's triangles, a_i is the corresponding coefficient, and n is the number of nodes in the local support domain.

For the radial PIM, using radial basis functions augmented with polynomials, a field variable function $u(\mathbf{x})$ can be approximated as follows:

$$u(\mathbf{x})=\sum_{i=1}^{n}R_i(\mathbf{x})a_i+\sum_{j=1}^{m}P_j(\mathbf{x})b_j=\mathbf{R}^T(\mathbf{x})\mathbf{a}+\mathbf{P}^T(\mathbf{x})\mathbf{b}, \tag{6}$$

where $R_i(\mathbf{x})$ and $P_j(\mathbf{x})$ are radial basis functions and polynomial basis functions, respectively, a_i and b_j are corresponding coefficient, n is the number of nodes in the local support domain used for constructing RPIM shape functions and m is the number of polynomial terms augmented to the radial basis functions [6].

The coefficients in Eqs. (5) and (6) can be determined by enforcing the field function to be satisfied at the n nodes within the local support domain. Finally, the PIM shape functions can be obtained and a displacement component can be interpolated as

$$u(\mathbf{x})=\boldsymbol{\Phi}^T(\mathbf{x})\mathbf{U}_s, \tag{7}$$

where $\boldsymbol{\Phi}(\mathbf{x})$ is the row-matrix of PIM shape functions and $\mathbf{U}_s$ is the vector of nodal displacement [6].

When linear polynomial PIM shape functions are used, four vertexes of the background four-node tetrahedron cell are taken to perform the interpolation of the interest points located inside the cell, which is same as the linear FEM does. This can be easily implemented and can always ensure the invertibility of the moment matrix, as long as the four vertexes of the tetrahedron are not on a plane. When radial PIM shape functions are used, we use eight nodes to perform the approximation: four vertexes of the cell plus four nodes located at the remote vertices of the four neighboring cells. For a boundary cell with three neighboring cells, four vertexes of the cell, three vertexes of its neighboring cells and another closet to the point of interest will be selected for constructing the RPIM shape functions.