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Computation of Hermite interpolation in terms of B-spline basis using polar forms ☆

A. Lamnii [a], M. Lamnii [b,*], F. Oumellal [a]

[a] Univ. Hassan 1st, Laboratoire de Mathématiques, Informatiques et Sciences de l'Ingénieur (MISI), Faculty of Sciences and Technology, Settat, Morocco

[b] Univ. Hassan 1st, Laboratoire d'Analyse et Modélisation des Systèmes et Aide à la Décision (LAMSAD), Ecole Supérieure de Technologie, Berrechid, Morocco

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Abstract

The aim of this paper is to present an Hermite interpolation problem with B-splines of high degree of smoothness. More precisely, we use polar forms to find the B-spline control points for Hermite interpolation. The resulting formula is used to give Hermite basis functions. In particular, Quadratic $\mathcal{C}^1$ and cubic $\mathcal{C}^2$ interpolations with sharp parameters are analyzed.

Keywords: B-splines; Hermite interpolation; Polar forms

1. Introduction

B-splines are fundamental to approximation and data fitting and have gained a lot of attention since they were introduced in [2]. Generally the representation of piecewise-polynomial functions in B-spline form has several desirable properties for geometric modeling. However, it is not always easy to express the interpolant in terms of a B-spline basis.

In the classical Hermite interpolation problem we are given a set of breakpoints $x_0 = a < \cdots < x_n = b$ and a set of interpolation values $c_{i,j} \in \mathbb{R}$, $i = 0, \ldots, n$, $j = 0, \ldots, k-1$, and we need to find a $\mathcal{C}^{k-1}$ polynomial function s of degree $2k-1$ in every interval $[x_i, x_{i+1}]$, such that $s^{(j)}(x_i) = c_{i,j}$. Such problems have been widely studied in the literature. Starting with the seminal work by Schoenberg [14], many authors have worked in this direction. For brevity, the reader is referred to [3,6,8,10,11,15,14,18]. In [11], Mummy has derived an explicit formula of B-spline control points for Hermite interpolation in terms of the interpolation data $c_{i,j}$. The author has used the de Boor–Fix dual functionals as an effective tool for solving this problem. The same approach has been used in [8] to deal with a problem having discontinuous derivatives of some order at some breakpoints. In [17,18], Seidel gave another simple and elegant proof of Mummy's result using polar forms.

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* Corresponding author.
E-mail addresses: a_lamnii@yahoo.fr (A. Lamnii), m_lamnii1@yahoo.fr (M. Lamnii), fatimaoumellal5@gmail.com (F. Oumellal).

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The purpose of this paper is to show how polar forms can also be used as a powerful tool for producing B-spline control points for Hermite interpolation in a spline space $\mathcal{S}_k^{k-1}(\tau, [a, b])$, where τ is a subdivision of $[a, b]$ obtained by adding some additional knots between the interpolation points $x_i, 0 \le i \le n$. There are several reasons for considering Hermite interpolation in this spline space. First of all, it provides a high degree of smoothness with low order. Moreover, another good reason, the added knots can be chosen to preserve certain geometric shape properties of the data such as monotonicity or convexity (see [6,10,12,15]).

The paper is organized as follows. In Section 2, we briefly introduce the problem and some preliminary knowledge about B-splines. In Section 3, we recall the definition and some basic properties of polar forms. More details can be found in [7,13,17,19]. In Section 4, after some necessary preliminaries on the adopted notation, we give the main results and we show how to express Hermite interpolation in B-splines form. In Section 5, as an application of the obtained results, we express the Hermite basis in terms of B-splines. Section 6 is devoted to the analysis of Hermite interpolation in the $\mathcal{C}^1$ quadratic and $\mathcal{C}^2$ cubic cases.

2. Preliminary of B-splines

Let $k,\ n \ge 2$ be given positive integers, and let $x_0 = a < x_1 < \cdots < x_n = b$ be a set of breakpoints on a given interval $[a, b]$. For any set of values $c_{i,j} \in \mathbb{R}$, $i = 0, \ldots, n$, $j = 0, \ldots, k-1$, to be interpolated, we want to find a piecewise polynomial function $s \in \mathcal{S}_k^{k-1}(\tau, [a, b])$ such that

$$s^{(j)}(x_i) = c_{i,j}, \quad i = 0, \ldots, n,\ j = 0, \ldots, k-1,$$

where τ is a partition of $[a, b]$ obtained by adding some new knots between the interpolation points $x_0 = a < x_1 < \cdots < x_n = b$. To solve this problem we first need to show that the dimension of space $\mathcal{S}_k^{k-1}(\tau, [a, b])$ must be equal to the number of parameters $c_{i,j}$, $i = 0, \ldots, n$, $j = 0, \ldots, k-1$. For this purpose, we just added $(n+1)(k-1)$ knots. To satisfy these requirements, foremost we add two simple knots x_{-1} and x_{n+1} such that $x_{-1} < x_0$ and $x_n < x_{n+1}$ at either end of the interval $[a, b]$. Then, in each subinterval $[x_i, x_{i+1}]$, $i = -1, \ldots, n$, we added $k-1$ distinct knots.

In the following, we denote the knots of partition τ by

$$x_{i,j}, \quad i = -1, \ldots, n,\ j = 0, \ldots, k-1 \text{ where } x_{i,0} = x_i.$$

Let $B_{i,k}$ be a univariate normalized B-spline of degree k associated with τ whose support interval is $[t_i, t_{i+k+1}]$, where

$$t_{ki+j} := x_{i,j}, \quad i = -1, \ldots, n,\ j = 0, \ldots,\ k-1. \tag{1}$$

This B-spline can be defined [5,4] by the recurrence relation

$$B_{i,k}(x) = \omega_{i,k}(x) B_{i,k-1}(x) + (1 - \omega_{i+1,k}(x)) B_{i+1,k-1}(x), \tag{2}$$

where

$$\omega_{i,k}(x) := \frac{x - t_i}{t_{i+k} - t_i}, \tag{3}$$

and

$$B_{i,0}(x) := \begin{cases} 1, & \text{if } t_i \le x < t_{i+1}, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

Every spline s in $\mathcal{S}_k^{k-1}(\tau, [a, b])$ has a unique representation

$$s(x) = \sum_{i=-k}^{kn-1} a_i\ B_{i,k}(x).$$

The values a_i are called the B-spline coefficients of s.

Once we have calculated the B-splines basis for a fixed integer k, we set

$$\mathcal{B}_{(i,j)}^k(x) := B_{ki+j,k}(x) \quad \text{and} \quad a_{i,j}^k := a_{ki+j}.$$