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Nonlinear analysis of elastic space cable-supported membranes

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ABSTRACT

In this paper a solution method is presented for the coupled problem of elastic flat or space membranes supported by elastic flexible cables. Both membrane and cable undergo large deflections. Starting from the minimal surface the membrane is prestressed by imposed boundary displacements under the self-weight. Then an iterative procedure is employed, which consists in solving the membrane and the cable large deflection problems separately in each iteration step and checking the continuity of displacements and forces between membrane and cable. The procedure is repeated until convergence is achieved. Both membrane and cable problems are solved using the analog equation method (AEM). The displacements as well as the stress resultants are evaluated at any point of the membrane and the cable from the integral representations of the solution of the analog equations, which are used as mathematical formulae. Example problems are presented, for both flat and space membranes, which illustrate the method and demonstrate its efficiency and accuracy.

1. Introduction

Structural membranes play an important role in the engineering field in our days. The lightness of the structure, the ability to cover very large spans and the prefabrication facility are some of their constructional advantages. They are separated into two major categories, the air-supported membranes and the prestressed membranes. The air-supported membrane derives its structural integrity from the use of internal pressurized air to inflate the structural fabric envelope, so that air is the main support of the structure. However, any prestress can be applied to the membrane of the second category by stretching it from its edges (imposed boundary displacements) or by prestressed cables which support it (cable-supported). The analysis of prestressed membranes involves three steps [1] (i) form-finding, (i) prestress under self-weight and (iii) in-service loading. There are various techniques for the determination of the initial shape of the membrane (e.g. [2–5]). In this investigation the initial form of the membrane is determined by solving the minimal surface problem using the method presented in Ref. [6].

In this paper an iterative solution scheme to the coupled problem of elastic flat or space cable-supported membranes is presented. Starting from the minimal surface the membrane is prestressed by imposed boundary displacements on the boundary. In first instance the cables are assumed undeformed (fixed boundary) and the membrane problem is solved under self-weight and prestress. Then the external loading is applied on the deformed membrane and the membrane problem is solved again with fixed boundary taking into account the prestress forces from the previous step. The resulting reactions on the membrane boundary are applied with reversed sign on the cable as external loading and the cable problem is solved. Then the computed displacements of the cable are used as imposed boundary displacements for the membrane and the membrane problem is solved again under the in-service loading. The procedure is repeated until the displacement continuity conditions between cable and membrane are satisfied.

Both membranes and cables exhibit nonlinear behavior due to near zero flexural stiffness which renders them susceptible to large deformations. That is, such structures adapt their shape undergoing large deflections, in order to provide transverse components of the stress resultants to equilibrate the load. In the present analysis geometric nonlinearity is considered which result in nonlinear kinematic relations, while the strains are still small compared with the unity. A consequence of this is that the resulting differential equilibrium equations are coupled and nonlinear. For flat membranes the problem is less difficult and various solutions (analytical, approximate and numerical) are available in the literature [7]. For space membranes the analytical solutions are limited to axisymmetric membranes where the problem is highly simplified as it becomes one-dimensional. However, membranes of arbitrary shape encountered in realistic engineering problems can be analyzed only by numerical methods [1,8–11]. Additional related references can be found in Ref. [1] for space membranes and in Ref. [7] for flat. For the coupled cable-membrane problem the FEM has been used by several authors with various formulations. Haber et al. [12] have used a computer-aided design program for the design of cable reinforced

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membranes structures based on a nonlinear analysis described in Refs. [13,14]. Fujikake et al. [15] investigated the nonlinear analysis of fabric tension cable structures using an updated Lagrangian formulation to include the large displacements. Tabbarrok and Qin [16] presented a complete procedure for both form finding and load analysis of tension structures as a combination of membranes, cables and frames. All these investigators, however, have presented their results in graphical form and hence their accuracy cannot be validated.

In this investigation both membrane and cable problems are solved using the analog equation method (AEM). According to this method the three coupled nonlinear second order partial differential equations in terms of the displacements describing the response of the membrane are replaced with three uncoupled Poisson's equations subjected to fictitious sources, unknown in the first instance, under the same boundary conditions. Subsequently, the fictitious sources are established using a procedure based on the BEM [1]. A similar procedure is applied for the solution of the cable problem undergoing large displacements. The three coupled nonlinear ordinary differential equations in terms of the displacements describing the response of the cable are replaced with three linear string equations under unknown fictitious loads, which are subsequently established using the integral equation method [17]. The final displacements due to the in-service loading as well as the stress resultants are evaluated at any point of the membrane and the cable from integral representations of the solution of the substitute problems, which are used as mathematical formulae. Example problems are presented, for both flat and space membranes, which illustrate the method and demonstrate its efficiency and accuracy.

Finally, the developed method can be combined with the domain decomposition method for nonlinear membranes [9] and offer an efficient computational tool for analyzing cable-supported membranes of complicated geometry encountered in realistic engineering tensile structures.

2. Problem statement and governing equations

2.1. The membrane problem

Consider a thin flexible elastic space (non-flat) membrane consisting of homogeneous linearly elastic material, whose reference configuration in its undeformed state is represented by the surface S, bounded by the space curves C (see Fig. 1). The equation of the surface S in Cartesian coordinates is $z=z(x,y)$. The surface S is projected on the xy plane, creating the domain Ω, bounded by Γ, the projection of C (see Fig. 1). The membrane, under the combined action of the arbitrarily distributed load p_x, p_y and p_z (acting along the Cartesian axes), is moved in its deformed configuration which is defined by the three displacement components $u=u(x,y)$, $v=v(x,y)$ and $w=w(x,y)$ along the Cartesian axes x, y and z.

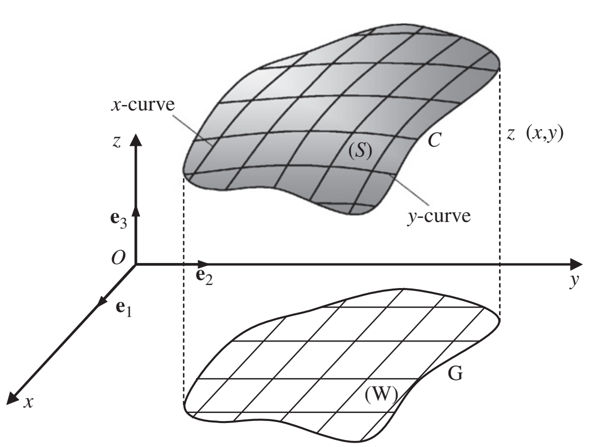

Fig. 1. Surface S and its projection domain Ω.

The equilibrium equations for the space membranes in terms of the displacement components have been derived in Ref. [1] and are given as

$$\begin{aligned}&\Big[C_{11}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{12}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{13}\big(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y}\big)\Big]_{,x}\\&\quad+\Big[C_{13}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{23}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{33}\big(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y}\big)\Big]_{,y}=-p_x\end{aligned}\tag{1a}$$

$$\begin{aligned}&\Big[C_{13}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{23}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{33}\big(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y}\big)\Big]_{,x}\\&\quad+\Big[C_{12}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{22}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{23}\big(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y}\big)\Big]_{,y}=-p_y\end{aligned}\tag{1b}$$

$$\begin{aligned}&\Big[C_{11}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{12}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{13}(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y})\Big](z_{,xx}+w_{,xx})\\&\quad+2\Big[C_{13}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{23}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{33}(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y})\Big](z_{,xy}+w_{,xy})\\&\quad+\Big[C_{12}\big(u_{,x}+z_{,x}w_{,x}+\tfrac{1}{2}w_{,x}^2\big)+C_{22}\Big(v_{,y}+z_{,y}w_{,y}+\tfrac{1}{2}w_{,y}^2\Big)\\&\quad+C_{23}(u_{,y}+v_{,x}+z_{,x}w_{,y}+z_{,y}w_{,x}+w_{,x}w_{,y})\Big](z_{,yy}+w_{,yy})\\&\quad=-p_z+p_x(z_{,x}+w_{,x})+p_y(z_{,y}+w_{,y})\end{aligned}\tag{1c}$$

in Ω, where C_{ij} are position dependent coefficients characterizing the stiffness of the membrane, given as

$$\begin{aligned}&C_{11}=D\frac{(1+z_{,y}^2)^2}{(1+z_{,x}^2+z_{,y}^2)^{3/2}}\\&C_{12}=D\frac{z_{,x}^2z_{,y}^2+\nu(1+z_{,x}^2+z_{,y}^2)}{(1+z_{,x}^2+z_{,y}^2)^{3/2}},\ C_{21}=C_{12}\\&C_{13}=-D\frac{(1+z_{,y}^2)z_{,x}z_{,y}}{(1+z_{,x}^2+z_{,y}^2)^{3/2}}\\&C_{22}=D\frac{(1+z_{,x}^2)^2}{(1+z_{,x}^2+z_{,y}^2)^{3/2}},\ C_{31}=C_{13}\\&C_{23}=-D\frac{(1+z_{,x}^2)z_{,x}z_{,y}}{(1+z_{,x}^2+z_{,y}^2)^{3/2}}\\&C_{33}=D\frac{2z_{,x}^2z_{,y}^2+(1-\nu)(1+z_{,x}^2+z_{,y}^2)}{2(1+z_{,x}^2+z_{,y}^2)^{3/2}},\ C_{32}=C_{23}\end{aligned}\tag{2}$$

and $D=Eh/(1-\nu^2)$ is the stiffness of the membrane with h being its thickness and E, ν the material constants.

Note that for $z_{,x}=z_{,y}=0$ Eqs. (1) yield the equilibrium equations of the flat membrane [7].

The pertinent boundary conditions of the problem are [1]

$$T_x=\tilde{T}_x \ \text{ or } \ u=\tilde{u},\tag{3a}$$

$$T_y=\tilde{T}_y \ \text{ or } \ v=\tilde{v},\tag{3b}$$

$$T_x(w_{,x}+z_{,x})+T_y(w_{,y}+z_{,y})=\tilde{T}_z \ \text{ or } \ w=\tilde{w},\tag{3c}$$

on Γ. The tilde over a symbol designates prescribed quantity. T_x, T_y are the components of the boundary tractions in Cartesian coordinates given as

$$T_x=N_x\cos\theta+N_{xy}\sin\theta,\tag{4a}$$

$$T_y=N_{xy}\cos\theta+N_y\sin\theta,\quad \theta=\measuredangle x,\mathbf{n}\tag{4b}$$