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Harmonic analysis of direct digital control of voltage inverters

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Abstract

Voltage inverters are the most common kind of actuator to interface with variable frequency electrical systems. They are usually driven by a modulation block that provides the gating signals for the switches. Direct digital control avoids the modulation block, being the gating signals directly produced by the controller. In such configuration, the control of the electrical system is a discrete-time and quantized-actuation problem. This paper analyzes the harmonic content in sinusoidal steady state resulting from such configuration. The analysis illustrates the higher harmonic content, providing lower bounds for such content in the case of low ratios of commutation to fundamental frequencies. Also, the dependence of harmonic content with duty cycle and with commutation losses is exposed. The findings apply to different kind of control structures, regardless of its design or tuning parameters. Some of them are experimentally compared. A two-level three-phase inverter is used in the analysis for its relevance and simplicity although the procedure can be applied to any number of phases or levels.

Keywords: Inverters; Discrete time control; Harmonic; Steady state

1. Introduction

The use of Voltage Source Inverters (VSI) requires generating gating signals for the semiconductor switches. In most cases this is accomplished by using a modulating device realizing techniques such as Pulse Width Modulation (PWM). In addition to the modulating devices an outer controller is typically used for reference tracking and disturbance rejection. The modulating component driving the VSI aims at providing chopped voltages with a fundamental component of a given frequency and amplitude which, in turn, imprints on the electrical system the desired electrical variables such as fluxes and currents. The modulating element acts as an interface for the controller, allowing the variables to appear (on the controller side) as time-continuous albeit they are not. In a very different setting, control commands can be applied directly to the VSI, without intermediate modulation blocks. Then the working scenario changes drastically; in particular, just a single voltage vector is used for the whole duration of the sampling period. This type of control is clearly the opposite of methods that modulate via pulse width, where the switches of the inverter can be changed at any time [12].

Recently, direct digital control of VSI have been used within applications of model based predictive controllers for different applications such as multi-phase Induction Machines (IM) [9], permanent magnet synchronous motors [7]

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and other electrical systems. It is seldom noted that digital control of VSI without modulation blocks lies in the realm of discrete-time and quantized-actuation control systems, and must be treated accordingly. In fact, the problem of harmonics produced by direct digital control of VSI has been little studied. In [10] a space vector modulation algorithm for five-phase open-end winding drive with minimum low-order harmonic content is developed. The superior harmonic performance is due to the use of a higher number of different voltage vectors. In [16] an instantaneous current control technique to compensate harmonics generated by non-linear load is developed. Selective compensation of main voltage harmonics in a grid-connected micro-grid is achieved through proper control of distributed generators interface converters in [18]. A study of the influence of predictive model accuracy can be found in [2]. It is worth remarking that sub-harmonics is an aspect rarely encountered in the direct digital control literature, existing some papers related to digital implementation of PWM [14] algorithms. Sub-harmonics are nevertheless of importance as they produce currents that can damage an induction machine quicker than other causes such as voltage imbalance [6], because of the heat generated by losses [8].

This paper analyzes the harmonic content of discrete-time control of VSI as a continuation of the findings in [1]. The analysis of harmonic content will be carried out considering the set of all sequences that can be issued by the controller in a fundamental period in steady state. There are very few related works. For instance, in [3] a Lagrangian formulation for determining optimal waveforms phase currents during fault is used. Although this paper does not deal with harmonic elimination, this subject is relatively similar. Selective harmonic elimination deals with the reduction or total elimination of some specific harmonics using appropriate control schemes and/or hardware (see [13] and references therein). Classic approaches deal with the selection of switching times for applications that require low switching frequency [15,4]. In harmonic reduction the goal is to reduce the total harmonic distortion and/or related figures of merit. For instance in [21] an analysis is made at each individual frequency to develop a droop controller adding harmonic voltage to the inverter reference and compensate the harmonic at output. In [19] a d-q voltage control is developed in conjunction with harmonics reduction in the output of stand-alone single-phase inverters by means of special observers. Finally, a recent paper [20] analyzes unbalanced conditions to provide harmonic current for voltage regulation including harmonic selective elimination.

The research literature regarding harmonics for systems where the VSI is directly commanded by a digital controller is scarce. For analog (or quasi-analog) realizations there are some works, for instance harmonics in Direct Torque Control (DTC) drives are analyzed in [11]. Recently, analytical closed-form expressions for harmonic distortion in PWM techniques have been given [5]. Also for devices simpler than a VSI there are analytical (i.e. closed form) expressions can be derived for the case of a diode bridge rectifier as the firing pattern is fixed [17].

It is worth remarking that the findings of the paper apply to any control structure that uses a single space vector during a switching period. The findings apply to different kind of control structures, regardless of its design or tuning parameters. A two-level three-phase inverter is used in the analysis for its relevance and simplicity although the procedure can be applied to any number of phases or levels. The relevance is clear in applications where energy performance is foremost.

2. Problem statement

The setting being investigated is a variable frequency electrical system driven by a VSI under direct digital control as depicted in Fig. 1. Please notice that said Fig. 1 presents a very general framework and many different control schemes fit into it. For the harmonic analysis presented here it is required that the controller issue a control action $u(k)$ every sampling period k. The actuation might be made according to available measurements, objectives, and its internal logic. Measurements are typically currents and mechanical speed, although this is irrelevant for the analysis that will be presented later. The objectives of the controller might be current tracking, and/or speed tracking and/or torque production, once again this is irrelevant for the analysis as will be made clear later. In Fig. 1 the objective of the controller is represented by a certain desired (reference) trajectory $x^*(t)$ defined over some or all components of the state-vector x. For instance, in stator current control the reference are given in terms of stator current components, thus $x^*(t) = i_s^*(t)$.

To proceed further consider that the digital controller is implemented in the form of a control program. This is a sample–compute–actuate cycle of period T_s referred to as sampling period. The action taken at each discrete time k is a gating vector $u(k)$ that is applied to the VSI and hold for the whole T_s. In the particular case of a three-phase inverter, the gating vector takes the form $u = (K_a, K_b, K_c) \in \mathbb{B}^3$, being its components the state of the ideal switches for each