Discrete Optimization 25 (2017) 86–104

Contents lists available at ScienceDirect

Discrete Optimization

www.elsevier.com/locate/disopt

On the complexity of the separation problem for rounded capacity inequalities

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HIGHLIGHTS

- NP-Completeness of the separation problem of the Rounded Capacity inequalities.
- Variants of Subset Sum and Equicut Problems.
- Polynomial reductions of the separation problem of Rounded Capacity inequalities.
- Answer an open question.

ARTICLE INFO

Article history:
Received 15 May 2016
Received in revised form 17 February 2017
Accepted 19 February 2017
Available online 27 March 2017

Keywords:
Branch-and-Cut
Polyhedral combinatorics
Computational complexity
Capacitated Vehicle Routing Problem

ABSTRACT

In this paper, we are interested in the separation problem for the so-called rounded capacity inequalities which are valid for the CVRP (Capacitated Vehicle Routing Problem) polytope. Rounded capacity inequalities, as well as the associated separation problem, are of particular interest for solving the CVRP, especially when dealing with the two-index formulation of the CVRP and Branch-and-Cut algorithms based on this formulation. To the best of our knowledge, it is not known in the literature whether this separation problem is NP-hard or polynomial (see for instance Augerat et al. (1998) and Letchford and Salazar (2015)), and it has been conjectured that the problem is NP-hard. In this paper, we prove this conjecture. We also show that the separation problem for rounded capacity inequalities is strongly NP-hard when the considered solution belongs to a particular relaxation of the problem and the clients have all the same demand.

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1. Introduction

The Capacitated Vehicle Routing Problem (CVRP) is defined by an undirected graph $G = (V, E)$, a special node $s \in V$, called *the depot*, and $V \setminus \{s\}$ the set of clients. Each client $u \in V \setminus \{s\}$ is given a positive demand d_u. To satisfy the demands, we use a fleet of identical vehicles, each of capacity $Q \geq 2$. Every edge uv of the graph is assigned a positive cost $c(uv)$ representing the travel time between nodes u and v. Each client must be served by a single vehicle and no vehicle can serve a set of clients whose total demand exceeds

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http://dx.doi.org/10.1016/j.disopt.2017.02.003
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its capacity. Also, each vehicle must leave from and return to the depot. The CVRP is to find a set of vehicle routes such that all the clients are served and the total routing cost is minimum. The client demands d_u, for all $u \in V \setminus \{s\}$, and the vehicle capacity are assumed, w.l.o.g, to be positive integers. We also assume that $d_u \le Q$, for all $u \in V \setminus \{s\}$.

The CVRP is known to be strongly NP-hard, since it generalizes the *Traveling Salesman Problem*, and has many practical applications, including network design, routing and scheduling problems. The problem, and its variants, have received a lot of attention for several decades and still remains the subject of extensive research. Several integer programming formulations have been proposed for the problem (see for instance [1–5]). In [6], Cornuejols and Harche investigated the structure of the polytope associated with the so-called two-index formulation of the CVRP. In this paper, we are interested in this latter formulation and the associated capacity inequalities which are defined below.

Given a node set $W \subsetneq V$, the *cut* induced by W in the graph G is denoted by $\delta_G(W)$ and is the set of edges of G having one node in W and the other in $V \setminus W$. When $W = \{u\}$, we denote by $\delta_G(u)$ the cut induced by W. Also, for a node set $W \subseteq V \setminus \{s\}$, we let $BP(W)$ be the minimum number of vehicles needed to serve the client set W. Namely, $BP(W)$ is the optimal solution of a *Bin Packing Problem* where bins are of capacity Q, object set is W and each object $u \in W$ is of size d_u.

The CVRP can be formulated by the following integer linear program (see [2,4,5]). Let $x \in \mathbb{R}^E$ be an integer vector, where $x(uv)$ is the number of times the edge $uv \in E$ is used by a vehicle in a solution of the CVRP. For every edge set $F \subseteq E$, we let $x(F) = \sum_{e \in F} x(e)$. The *two-index formulation* for the CVRP is given by

$$\min \sum_{e \in E} c(e)x(e)$$

s.t.

$$x(\delta_G(u)) = 2, \quad \text{for all } u \in V \setminus \{s\}, \tag{1.1}$$

$$x(\delta_G(W)) \ge 2BP(W), \quad \text{for all } W \subseteq V \setminus \{s\}, \quad |W| \ge 2, \tag{1.2}$$

$$0 \le x(uv) \le 1, \quad \text{for all } uv \in E \text{ with } u, v \in V \setminus \{s\}, \tag{1.3}$$

$$0 \le x(su) \le 2, \quad \text{for all } u \in V \setminus \{s\} \text{ with } su \in E, \tag{1.4}$$

$$x(uv) \in \mathbb{Z}, \quad \text{for all } uv \in E. \tag{1.5}$$

Constraints (1.1) are the *degree constraints* and indicate that every client is served exactly once. Inequalities (1.2) are the so-called *capacity inequalities*. They ensure that the total demand of a set of clients does not exceed the capacity of the vehicle used to serve these clients. They also ensure that the routes of the vehicles are connected. Inequalities (1.3)–(1.4) are the *trivial inequalities* and constraints (1.5) are the *integrality constraints*. The polytope described by constraints (1.1)–(1.5) is the so-called *CVRP polytope*, and is denoted by $\text{CVRP}(G, d, Q)$. We also denote by $\text{CVRP}'(G, d, Q)$ the polytope defined by the linear relaxation of the above formulation. In [6], Cornuejols and Harche studied the structure of the CVRP polytope. They gave several properties of that polytope as well as conditions for the capacity inequalities (1.2) to define facets.

Several authors devised Branch-and-Cut algorithms based on the two-index formulation to solve the CVRP (see for instance [7,8,4,5]). Since in this formulation, inequalities (1.2) are exponential in number, one has to address the separation problem associated with $\text{CVRP}'(G, d, Q)$, in order to solve the linear relaxation of the CVRP using a cutting plane method. Recall that the *separation problem* associated with a polyhedron P is to say if a given solution $\overline{x}$ belongs to P or not, and if $\overline{x} \notin P$, then exhibit an inequality $ax \ge \alpha$ valid for P and violated by $\overline{x}$. We also define the separation problem associated with a solution $\overline{x}$ and a family $\mathcal{F}$ of inequalities, which consists in saying if $\overline{x}$ satisfies or not all the inequalities of $\mathcal{F}$, and if not, exhibit at least one inequality of $\mathcal{F}$ which is violated by $\overline{x}$. An algorithm solving a separation problem

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